Bayesian Inference and Integrated Data Analysis in Nuclear Fusion

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13th ITER International School 2024, Nagoya, Japan, December 9-13, 2024

My Background

1990	Master	Technical University, Munich, Germany
1993	PhD	Ludwig-Maximilian-University, Munich, Germany
-1996	Postdoc	Max-Planck-Institute for Plasma Physics, Garching, Germany
1996-	Staff	Max-Planck-Institute for Plasma Physics, Garching, Germany

EFDA deputy working group leader; chair of ITPA IDAV SWG; member of ITPA Diagnostics TG

1993 -	Data analysis using Bayesian probability theory (~340 Publications)
1999	Initiating Integrated Data Analysis (W7-AS stellarator)
2004 -	IDA at ASDEX Upgrade
2014 -	IDE kinetic equilibrium reconstruction coupled with current diffusion

ASDEX Upgrade: estimating profiles T_e, n_e, T_i, n_i, Z_{eff}, v_{tor} ; equilibrium reconstruction

Integrated Data Analysis for Nuclear Fusion

Different measurement techniques for the same quantities → redundant and complementary data

Coherent combination of measurements from different diagnostics for plasma control and physics studies

Goal:

- replace combination of results from individual diagnostics
- with combination of measured data
  → one-step analysis of pooled data

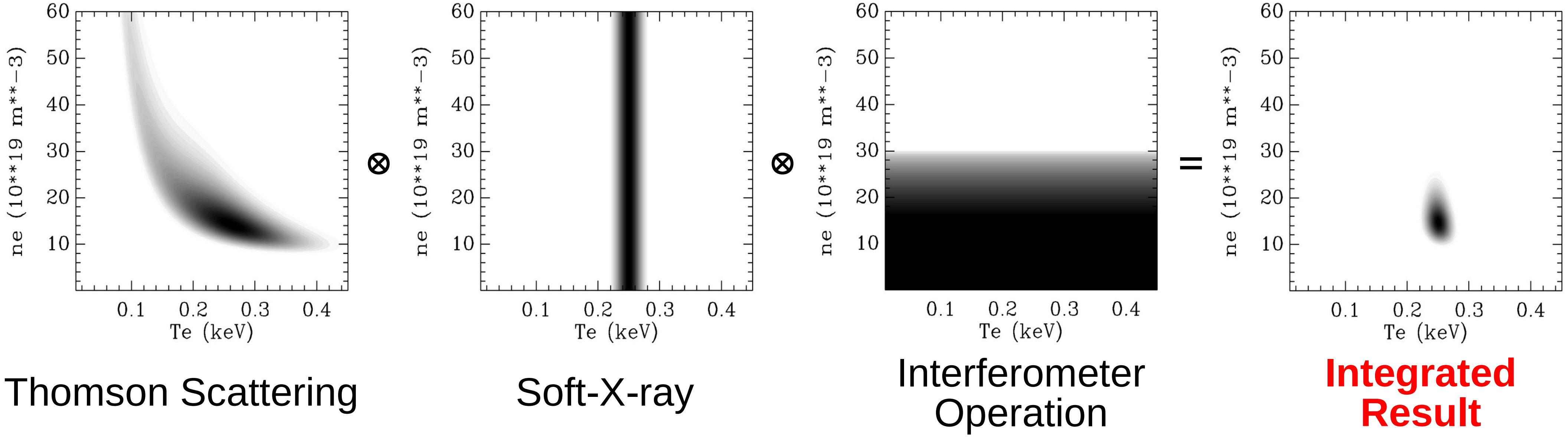
Tool:

- Bayesian probability theory

Integrated Data Analysis

Set of diagnostics to be combined

- n_e … electron density
- T_e … electron temperature



Probabilistic framework

R. Fischer, A. Dinklage, E. Pasch, PPCF 2002, PPCF 2003

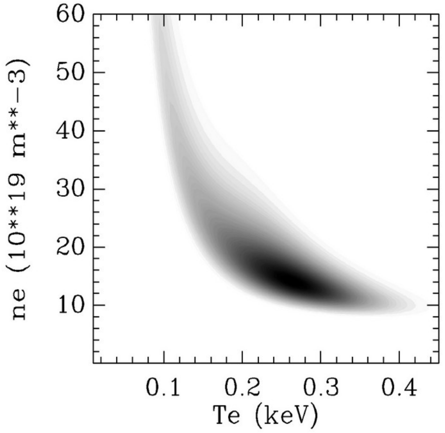
Synergistic effect by combined analysis

n_e, T_e: Thomson scattering and soft X-ray

R. Fischer, A. Dinklage, and E. Pasch, Bayesian modelling of fusion diagnostics, Plasma Phys. Control. Fusion, 45, 1095-1111 (2003)

Using synergism: *Set* of diagnostics to be combined

$\int$



Thomson Scattering

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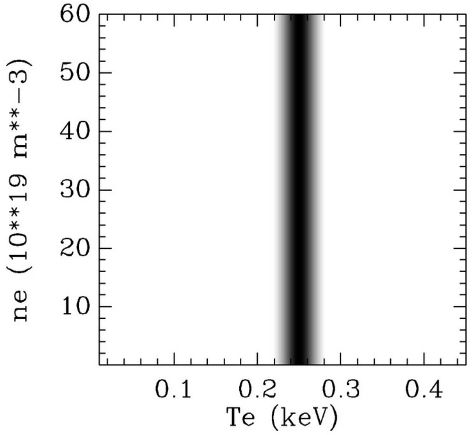
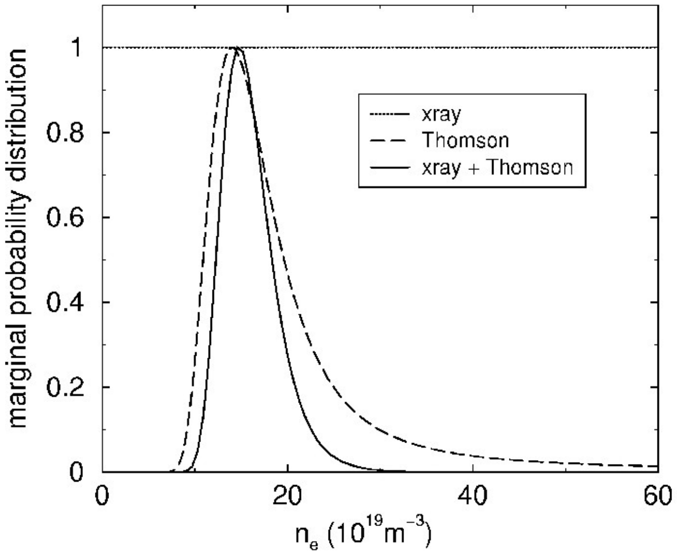


Soft-X-ray

dT_e =



Electron density

30% reduced error

→ synergism by exploiting full probabilistic correlation structure

Outline

- Synergistic effect using probability distributions
- Bayesian concept and recipe
- Conventional vs integrated data analysis
- Examples from ASDEX Upgrade
  - electron density and temperature profiles
  - ion temperature and rotation profiles
  - profile uncertainty
  - magnetic equilibrium reconstruction
- Example from ITER

Why Bayesian Probability Theory?

Scientific inference: prior information + new data ⇒ new knowledge

But: prior information and data are uncertain

⇒ new knowledge is uncertain

⇒ propositions (hypotheses) with a ***degree of truth***

⇒ quantification of ***degree of truth*** with probabilities

⇒ *Bayesian* probability theory

How to

- handle data uncertainties (statistic, systematic, outliers)
- handle (uncertain) nuisance parameters
- exploit prior information
- combine information / multiple data sets
- calculate with probability distributions
- estimate parameters and their uncertainties

History

- Thomas Bayes, 1702-1761, England

  minister of the Presbyterian Chapel in Tunbridge Wells

  „*Theory of probability in Essay towards solving a problem in the doctrine of chances*“, Philosophical Transactions of the Royal Society of London in 1764

  „Inverse problems“

- Pierre-Simon Laplace, 1749-1827, France

  accepted Bayes conclusions in 1781

  Théorie analytique des probabilités, 1812

  many applications: mortality, life expectancy, length of marriages, legal matters; errors in observations; the determination of the masses of Jupiter, Saturn and Uranus; triangulation

- Boole,1815-1865, England

  „*The Mathematical Theories of Logic and Probabilities*“

- Andrey Nikolaevich Kolmogorov, 1903-1987, Russia

  „*Analytic methods in probability theory*“

Interpretation of Uncertainties

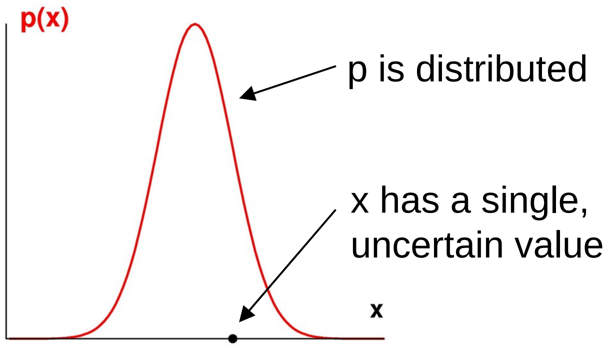
Bayesian: Probability describes uncertainty

Data:
- statistical (counting)
- measurement uncertainty (ruler)
- systematic (mis-alignment, mis-calibration)
- outliers (unknown or known cause)

Hypotheses:
- parameter of interest
- parameter of nuisance
- number of parameters
- physical models
- future data

p(x|I) describes how probability (plausibility) is distributed among the possible choices for *x* in the case at hand (information *I*)

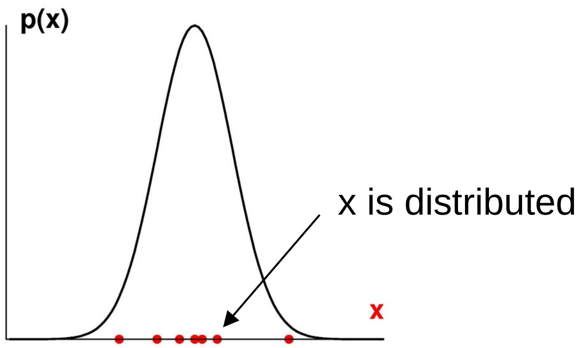
probability $\neq$ frequency



Frequentist: Probability describes ``randomness´´

p(x|I) describes how *x* is distributed throughout an infinite ensemble:

probability $\equiv$ frequency



Rules of Probability Theory

Bayesian: Probability describes knowledge conditional on information

Conditional Probability:

$P(A|B)$ ... Probability of proposition (statement, hypothesis) A given truth of proposition B.
quantification of uncertainty (degree of belief) of A („Bayesian“),
not frequency of outcomes of random variable A („Frequentist“)

Probability Theory Axioms:

- sum rule (OR) $P(A+B) = P(A) + P(B) - P(A,B)$

$\Longrightarrow$ $P(B) = \sum_i P(B, A_i)$ marginalization rule

- product rule (AND) $P(A,B) = P(A|B)\,P(B) = P(B|A)\,P(A)$

$\Longrightarrow$ $P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$ Bayesian Theorem

$$posterior = \frac{likelihood * prior}{evidence}$$

Clear instruction how to solve a problem:
If problem is well described than there is only one way to proceed

Example: Sum and Product Rule

Urn with w and b balls
with masses m and M:

$\alpha,\ \beta$	#	$p(\alpha,\beta)$
w, m	100	0.1
w, M	200	0.2
b, m	300	0.3
b, M	400	0.4
	1000	1.0

$$P(w) = P(w,m) + P(w,M) \qquad \text{sum rule}$$
$$= 0.1 + 0.2 = 0.3$$

What is the probability of having a *M* ball if I drew a *w* one?

$$P(M\,|\,w) = \frac{P(w,M)}{P(w)} \qquad \text{product rule}$$
$$= \frac{P(w,M)}{P(w,m)+P(w,M)} \qquad \text{sum rule}$$
$$= \frac{0.2}{0.1+0.2} = \frac{2}{3}$$

Bayesian Recipe for IDA: LIB + DCN + ECE + TS

Reasoning about parameter n_e, T_e:

(uncertain) prior information		$p(n_e, T_e)$	prior distribution
+ experiment 1:	$d_{LiB} = D_{LiB}(n_e, T_e) + \epsilon$;	$p(d_{LiB} \mid n_e, T_e)$	likelihood distributions
+ experiment 2:	$d_{DCN} = D_{DCN}(n_e) + \epsilon$;	$p(d_{DCN} \mid n_e)$	
+ experiment 3:	$d_{ECE} = D_{ECE}(T_e) + \epsilon$;	$p(d_{ECE} \mid T_e)$	
+ experiment 4:	$d_{TS} = D_{TS}(n_e, T_e) + \epsilon$;	$p(d_{TS} \mid n_e, T_e)$	

+ *Bayes theorem*

$$p(n_e, T_e \mid d_{TS}, d_{ECE}, d_{LiB}, d_{DCN}) \propto p(d_{TS} \mid n_e, T_e) \times p(d_{ECE} \mid T_e) \times p(d_{LiB} \mid n_e, T_e) \times p(d_{DCN} \mid n_e) \times p(n_e, T_e)$$

posterior distribution

+ additional uncertain (nuisance) parameter → *marginalization*

$$p(n, T \mid d) = \int p(n, T \mid d, \alpha)\, p(\alpha)\, d\alpha$$

generalization of Gaussian error propagation laws

Likelihood probability distribution

describes the uncertainty/statistics of data: $d_i = D(T, x_i, \alpha) + \epsilon_i$

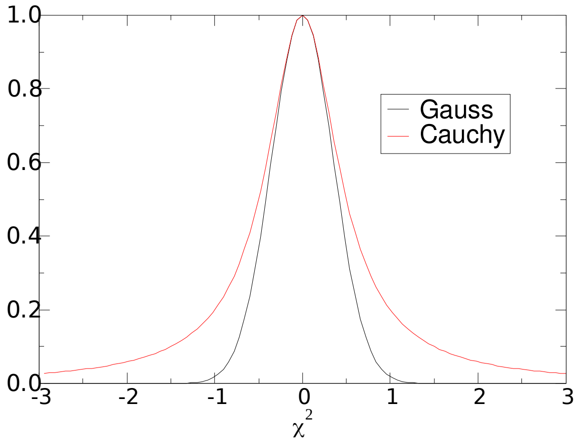
forward modeled (synthetic) data: $D(T, x_i, \alpha)$

noise (measurement uncertainty): ϵ_i

likelihood: $p(d_i|T) = p(\epsilon_i = d_i - D(T, x_i, \alpha))$

- Gaussian: for independent, normally distributed measurement errors with uncertainty σ $\quad \langle\epsilon\rangle = 0 \quad \langle\epsilon^2\rangle = \sigma^2$

$$p(\vec{d}|T, \vec{\sigma}) = \frac{1}{\prod_i^N \sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{\chi^2}{2}\right\} \quad \text{with} \quad \chi^2 = \sum_i^N \frac{[d_i - D_i(T)]^2}{\sigma_i^2}$$



- Cauchy, Student's-t: robust estimation (outliers, data failures)

  → workhorse in fusion data analysis

$$p(\vec{d}|T, \vec{\sigma}) \propto \prod_i^N \{a + \chi_i^2/2\}^{-(a+0.5)}$$

- Poisson: Counting experiments $\quad p(\vec{d}|D(\lambda, \vec{x})) = \prod_i^N \frac{D_i^{d_i}}{d_i!} \exp(-D_i)$

Prior probability distribution

Quantification of relevant additional information that is available independent of the data

Data Analysis - A Bayesian Tutorial, D. S. Sivia, Clarendon (1996)

- non-physical:
  - smoothness: Tikhonov, MaxEnt, min-Fisher information, ...
  - number of parameters, e.g. for profiles
  - correlation lengths, e.g. Gaussian process regression (GPR)

R. Fischer et al., Integrated Data Analysis and Validation, Chap. 10, NF, to be published
arXiv:2411.09270 [physics.plasm-ph]

- physical:
  - positivity constraints, e.g. n_e, T_e, n_i, T_i ; boundaries $Z_{eff} \geq 1$; monotonicity, moments
  - calibration measurements + **uncertainties**
  - data bases, atomic data including their **uncertainties**
  - predictive modeling
    - → parameters (T_e, n_e, T_i, v_{tor}, ...) are physically correlated
    - → example: transport codes providing profile (logarithmic) gradients or upper limits of gradients
      ASTRA-TGLF kinetic modeling *(M. Bergmann, Nuclear Fusion 64 (2024) 056024)*

To obtain most reliable results
1) exploit all (uncertain) information you have
2) quantify it with probability distributions
3) and multiply it to the likelihood

Bayesian Estimates

Posterior distribution: $$p(\theta|d) = \frac{p(d|\theta) \times p(\theta)}{p(d)}$$

Best estimates: $$max_\theta \; p(\theta|d) \rightarrow \hat{\theta}$$

$$mean_\theta \; p(\theta|d) \rightarrow \langle \theta \rangle$$

Uncertainties:

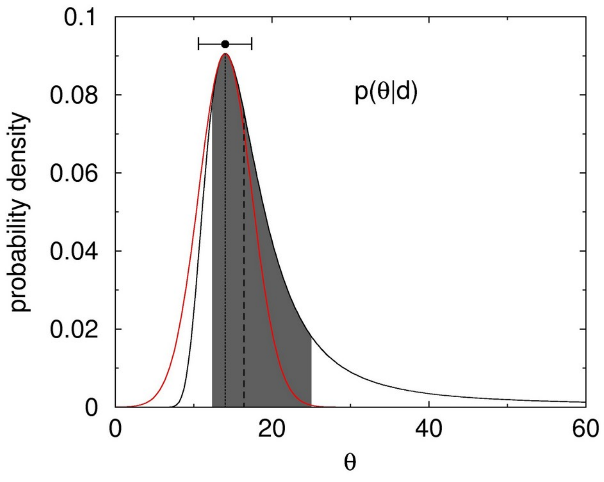
Laplace approx. $$var_\theta \; p(\theta|d)|_{\hat{\theta}} \rightarrow \sigma_\theta^2$$

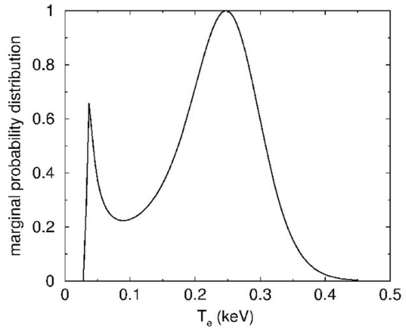
distribution variance $$var_\theta \; p(\theta|d) \rightarrow \Delta\theta^2$$

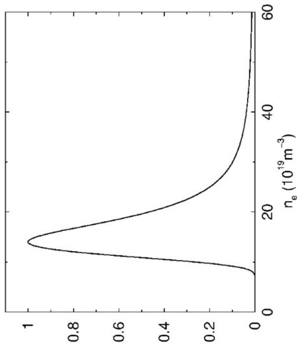
credibility (confidence) regions

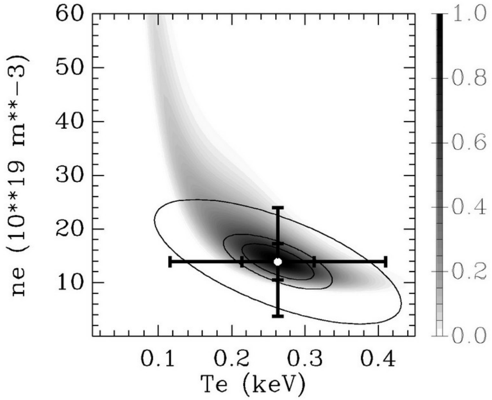
$68.3\%, 95.4\%, 99.73\%$ -*intervals*

(50±34)% percentile



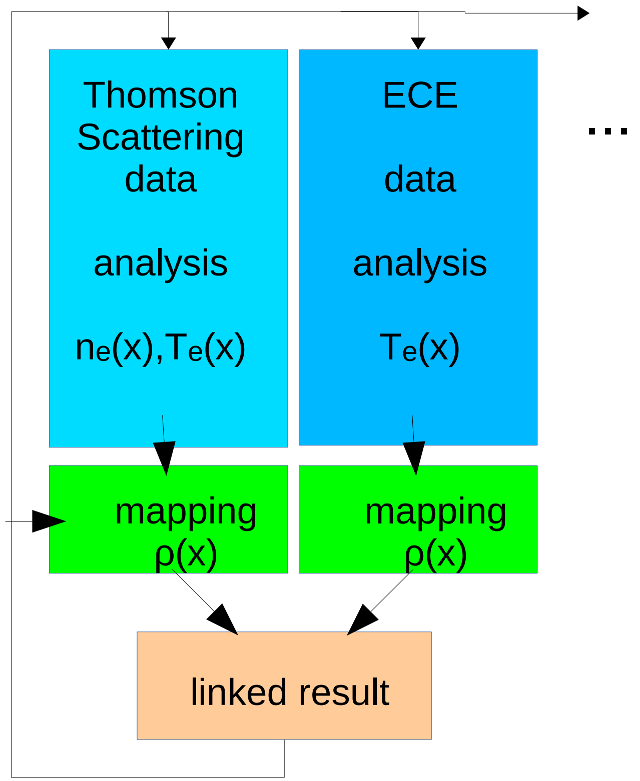






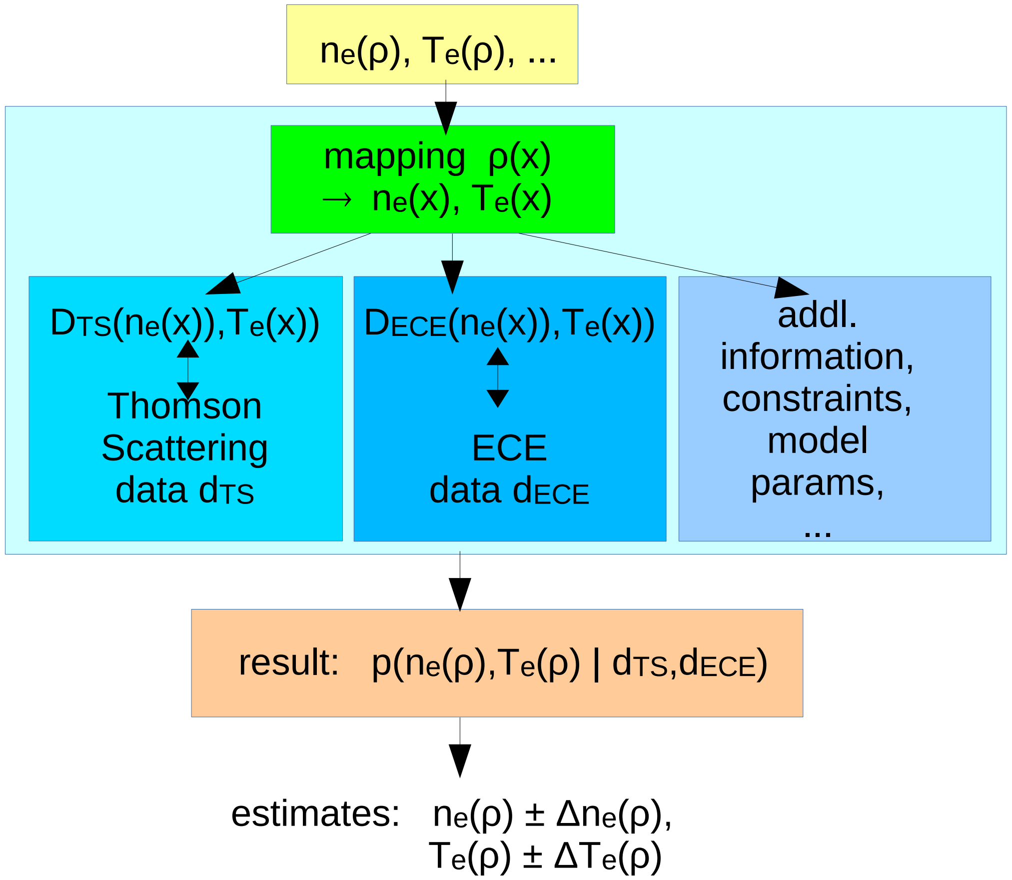
Conventional vs. Integrated Data Analysis

conventional



Parametric entanglements

IDA (Bayesian probability theory)



Conventional vs. Integrated Data Analysis (cont.)

Drawbacks of conventional data analysis: iterative

- (self-)consistent results? (cumbersome; do they exist?)
- difficult to be automated (huge amount of data from steady state devices: ITER, ...)
- information propagation? (Single estimates as input for analysis of other diagnostics?)
- error propagation? (frequently neglected: underestimation of the uncertainty)
- data and result validation? (How to deal with inconsistencies?)
- often backward inversion techniques (noise fitting? numerical stability?)
- result: estimates and error bars (sufficient? non-linear dependencies?)

Probabilistic combination of different diagnostics (IDA)

- ✔ uses only forward modeling (complete set of parameters → modeling of measured data)
- ✔ additional physical information easily to be integrated
- ✔ systematic effects (inconsistency) → describe with (nuisance) parameters
- ✔ unified error interpretation → Bayesian Probability Theory
- ✔ result: probability distribution of parameters of interest incl. all dependencies

IDA offers a unified way
of combining data (information) from various experiments (sources)
to obtain improved results

ASDEX Upgrade

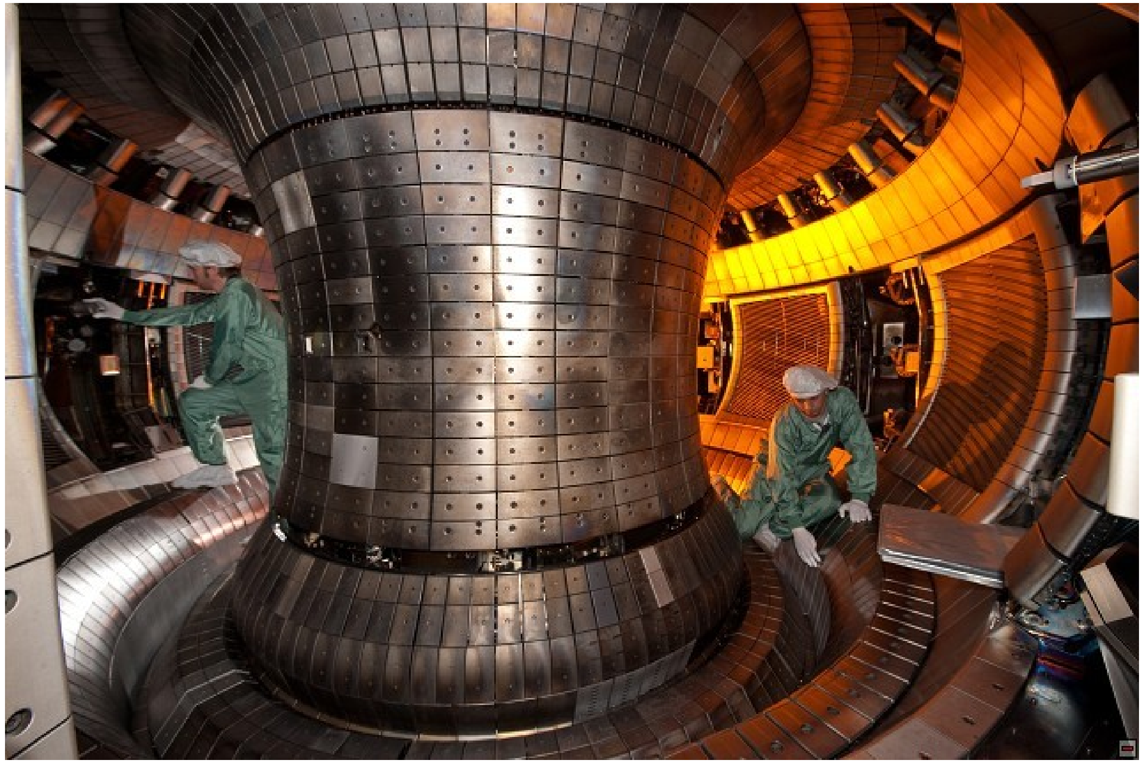
Axial **S**ymmetric **D**ivertor **EX**periment

ASDEX	1980-1990
ASDEX Upgrade	1990-

Garching, near Munich, Germany

mid-size tokamak (similar to DIII-D)

- major plasma radius 1.65 m
- minor plasma radius 0.5 m
- plasma volume 14 m^3
- max magnetic field 3.1 T
- plasma current 0.4 MA - 1.6 MA
- pulse duration 10 s
- plasma types: deuterium, hydrogen, helium
- plasma heating: 35 MW
  - ohmic 1 MW
  - neutral beam injection 20 MW
  - ion cyclotron 6 MW
  - electron cyclotron 8 MW for 10 s



Highlights:

- invention of the divertor
- discovery and description of the H-mode
- metallic wall

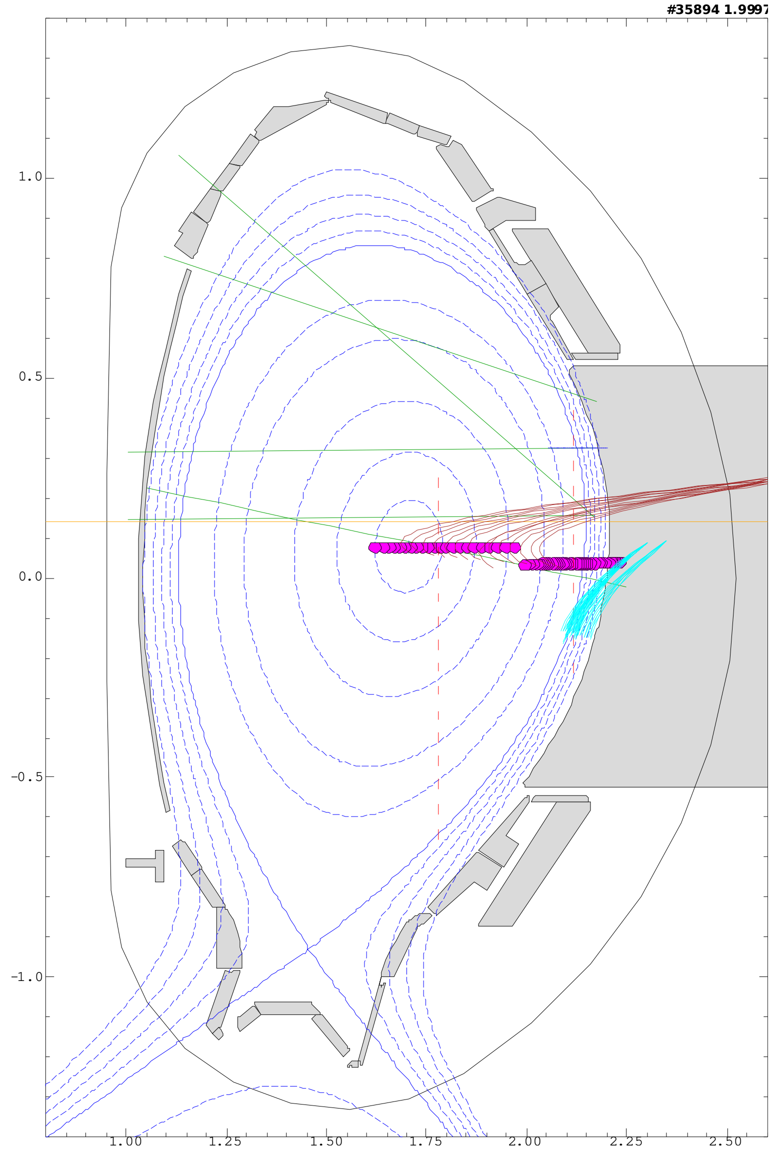
Application: ASDEX Upgrade

multi-diagnostic profile reconstruction: n_e , T_e

- Lithium beam impact excitation spectroscopy (LIB)
  collisional radiative model → $n_e (T_e)$
- Interferometry measurements (DCN) → n_e
- Electron cyclotron emission (ECE)
  ECRad: Electron cyclotron radiation transport → T_e (n_e)
- Thomson scattering (TS) → n_e , T_e
- Reflectometry → n_e
- Beam emission spectroscopy → n_e (Z_{eff})
- Thermal Helium beam spectroscopy → n_e , T_e

- Equilibrium reconstructions for diagnostics mapping
  (*IDE*: kinetic Grad-Shafranov solution coupled with current diffusion)

<u>A lot of dependencies and uncertainties:</u>
<u>We need a probabilistic approach!</u>



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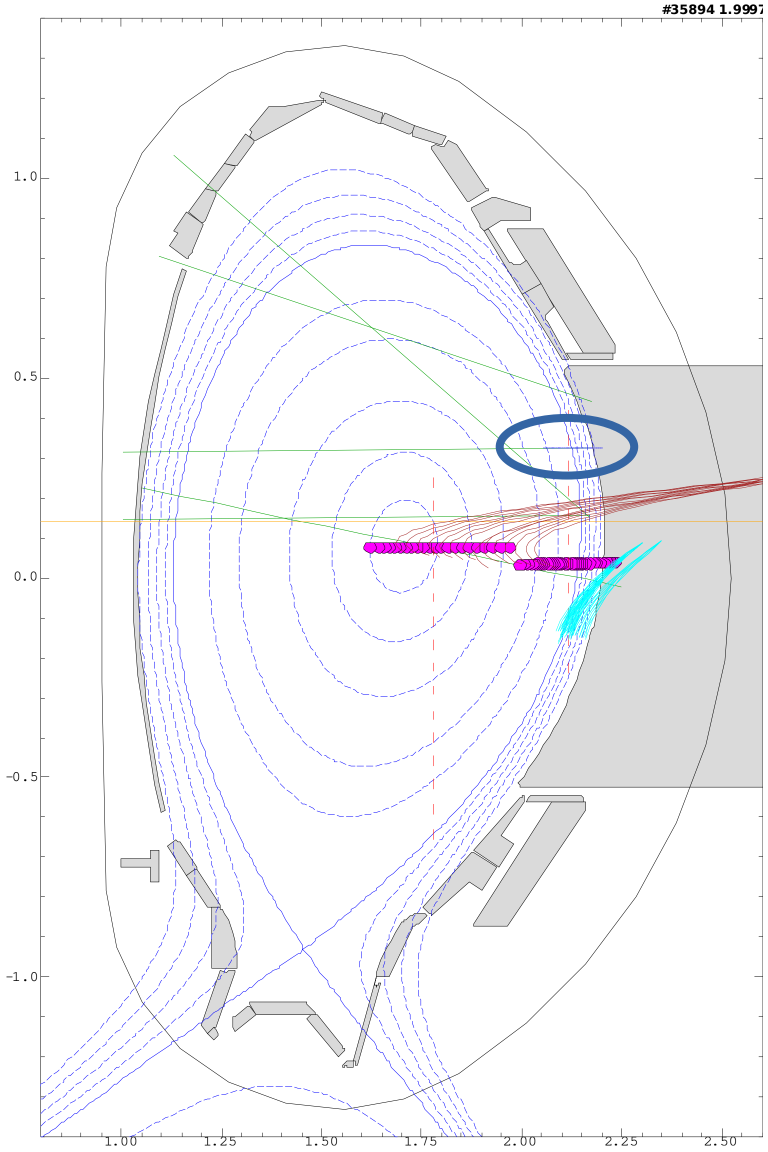


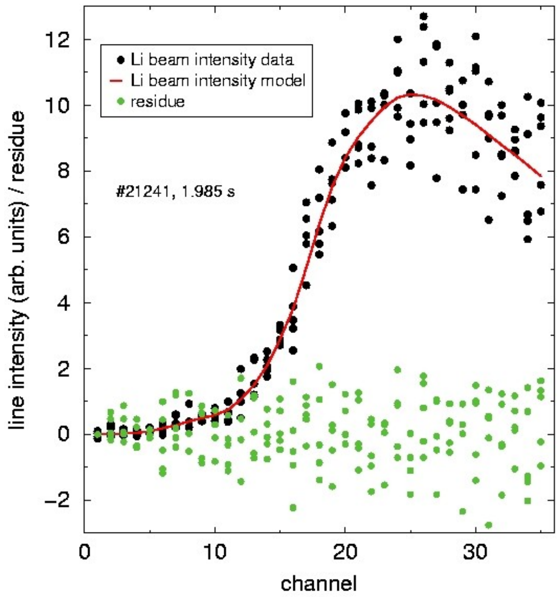
IDA: LIB

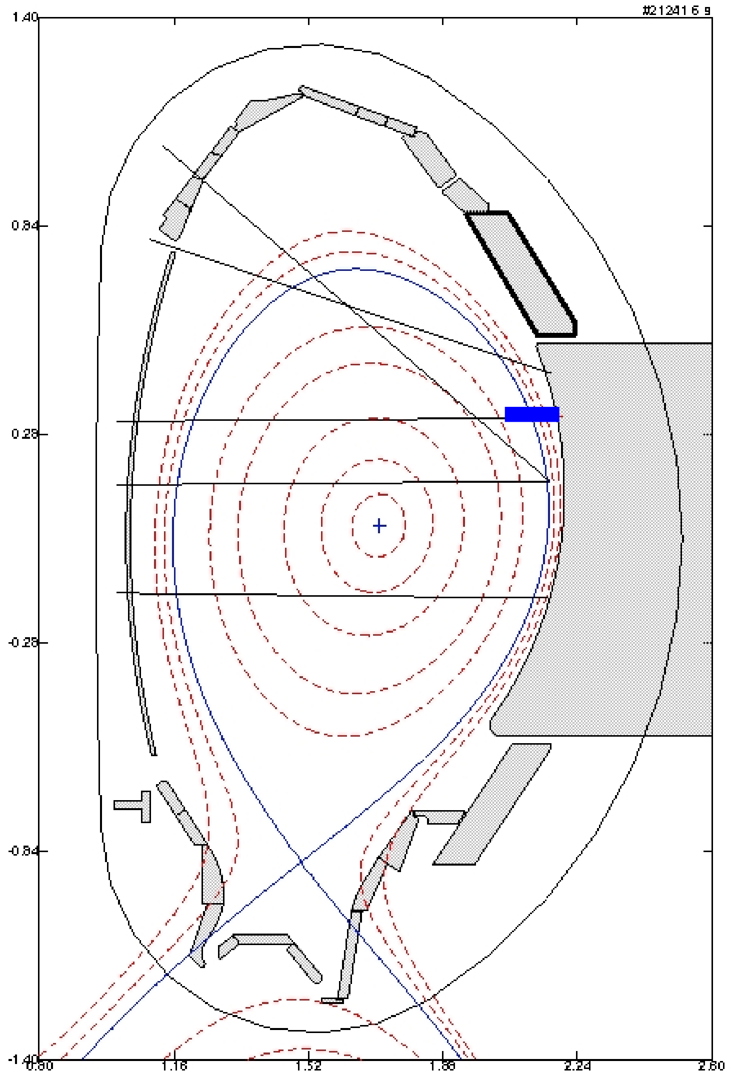
Lithium beam impact excitation spectroscopy

LiI radiation from neutral Lithium

E=30-80 keV

Li(2p) → Li(2s), l = 670.8 nm





Solve a collisional radiative model

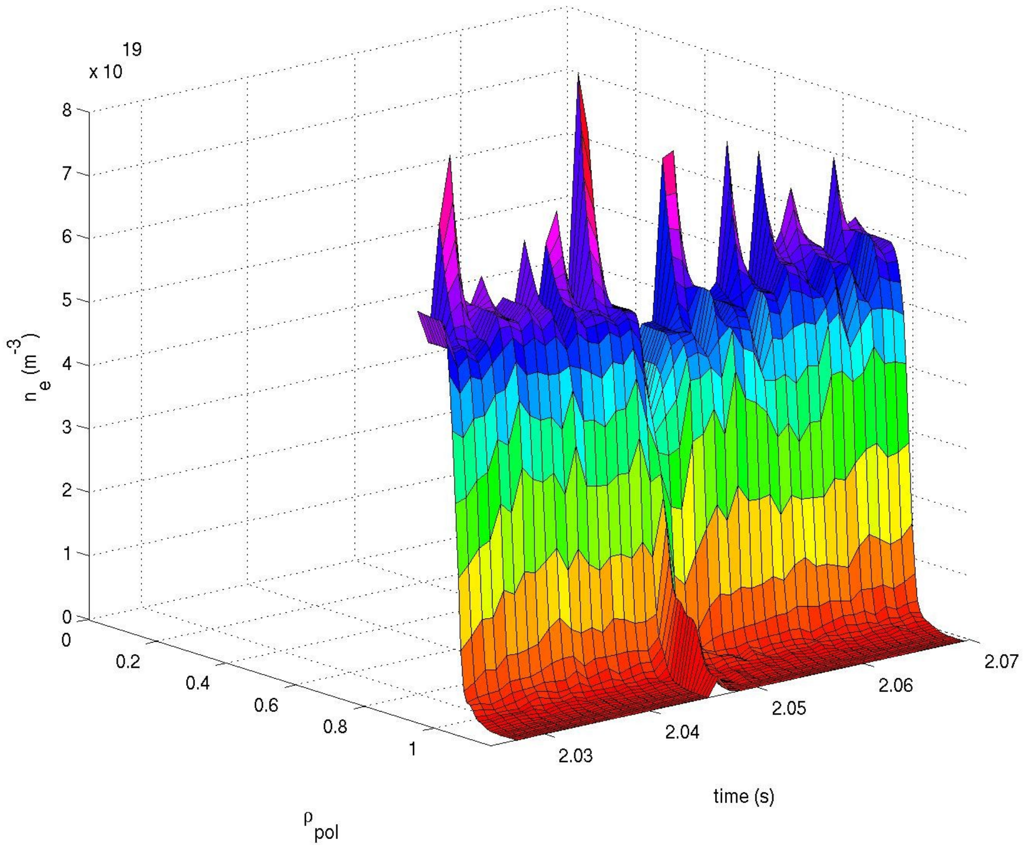
System of coupled linear differential equations:

$$\frac{dN_i(x)}{dx} = \sum_{j=1}^{N_{Li}} \{ n_e(x) a_{ij}(T_e(x)) + b_{ij} \} N_j(x) \quad ; \quad N_i(x=0) = \delta_{1i}$$

solved for a given profile $n_e(x)$ to obtain

occupation density Li(2p): $N_2(x|n_e)$

IDA: LIB



LIB: Lithium beam only

→ edge density profile

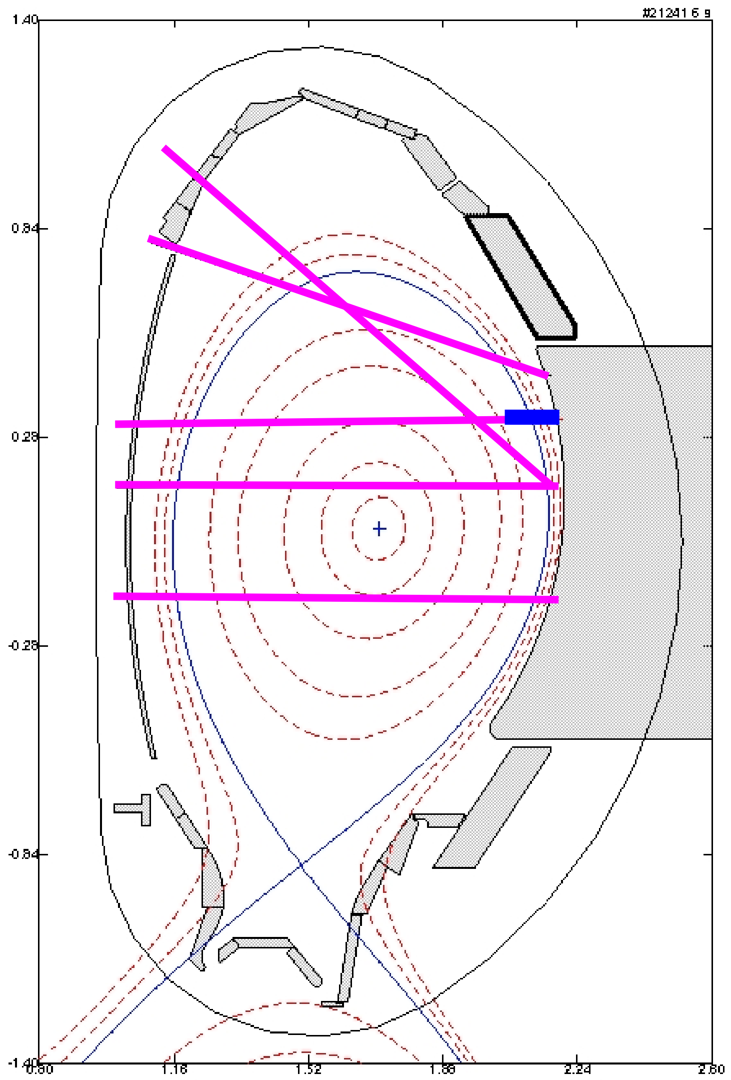
IDA: LIB + DCN Interferometer

Interferometry with DCN laser:

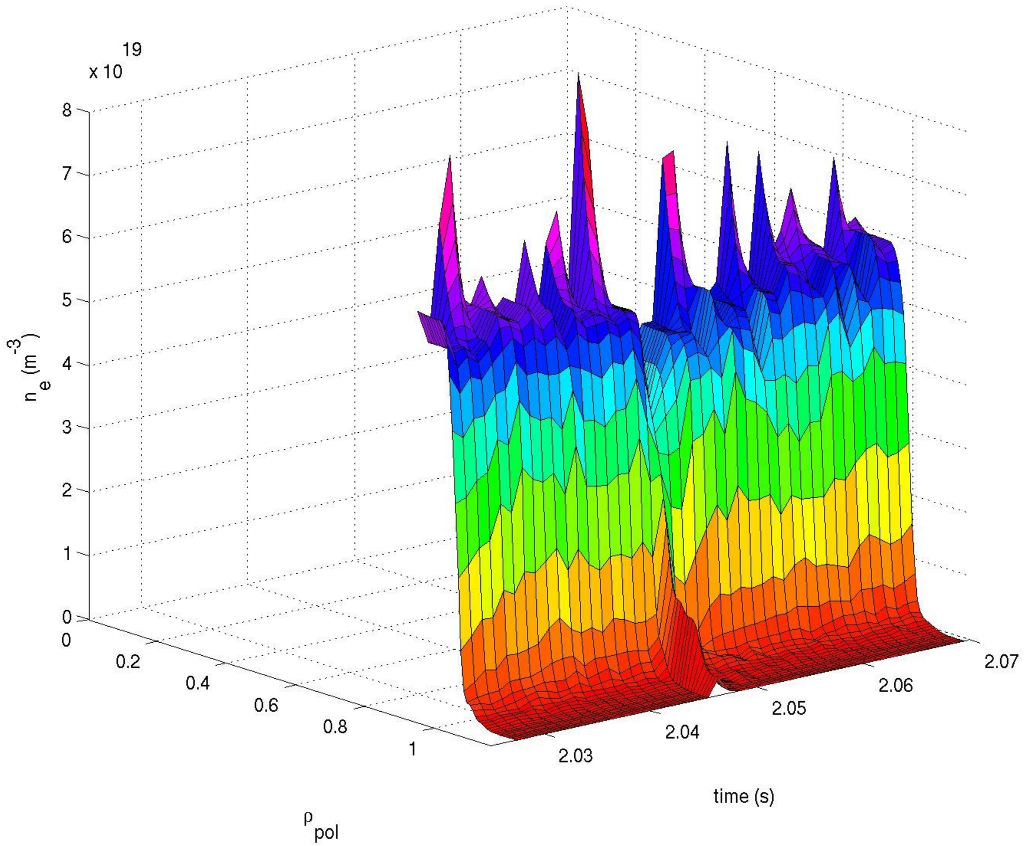
- Intensity proportional to line integrated electron density

$$d \propto \int n_e \, dl$$

- poor spatial resolution
- reliable absolute calibration



IDA: LIB + DCN



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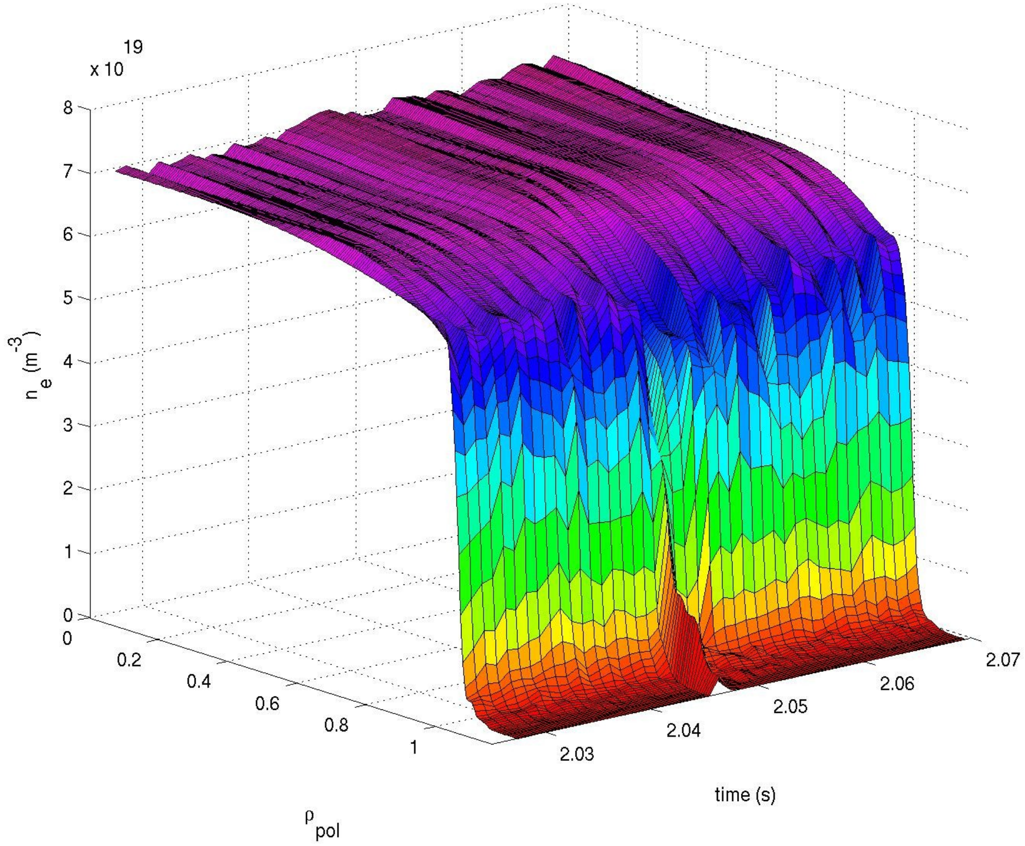
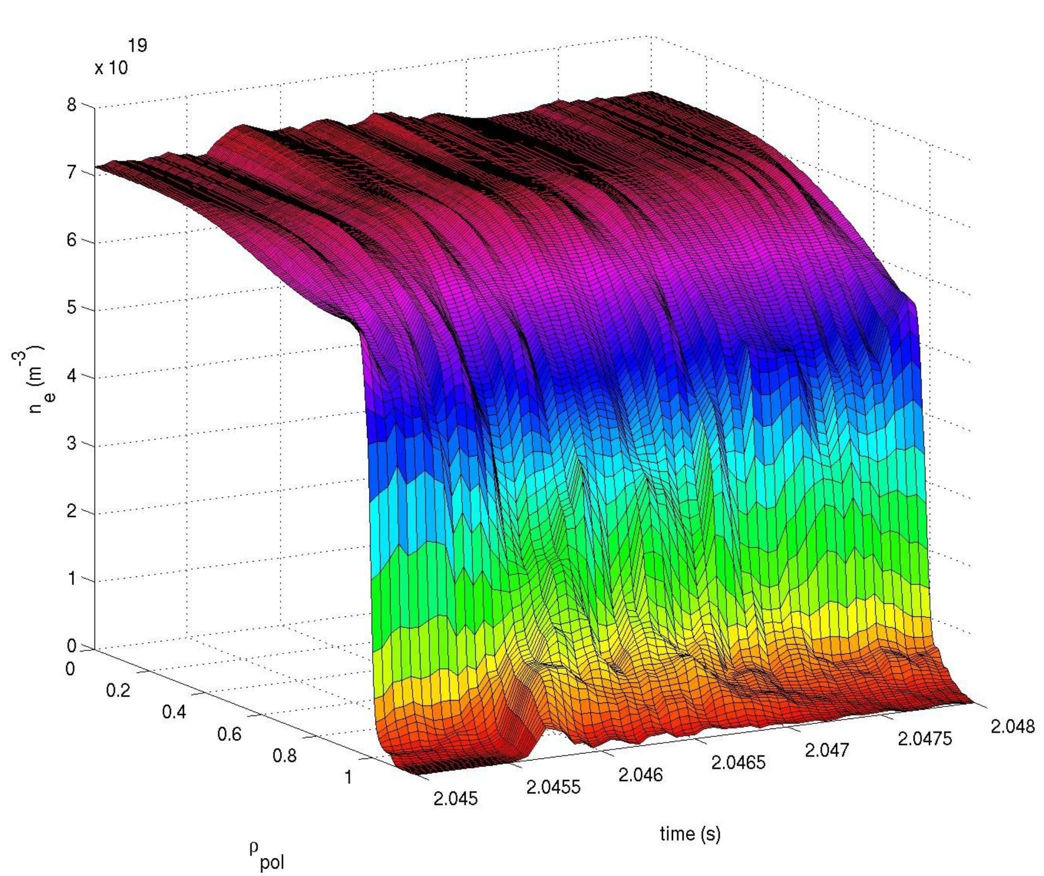
→ edge density profile

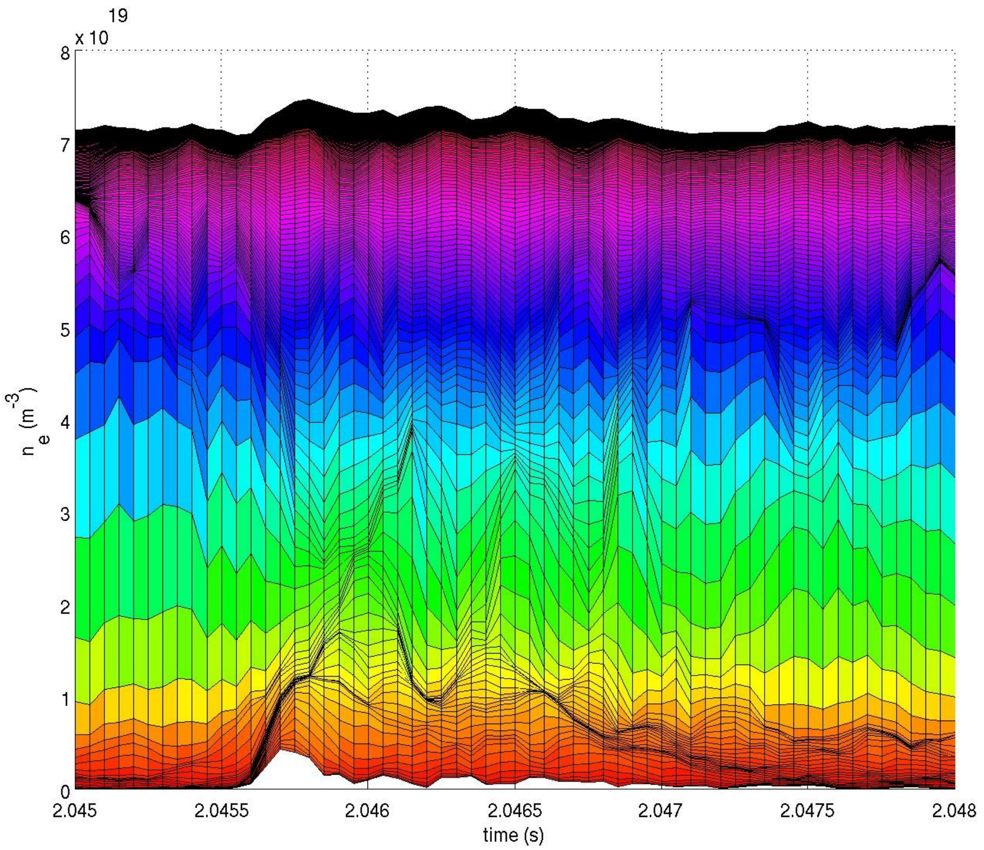


IDA: Lithium beam + DCN Interferometry

→ full density profile

IDA: LIB + DCN



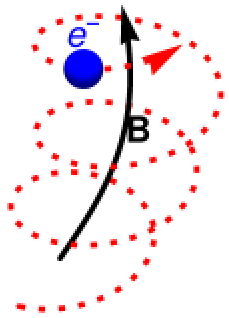


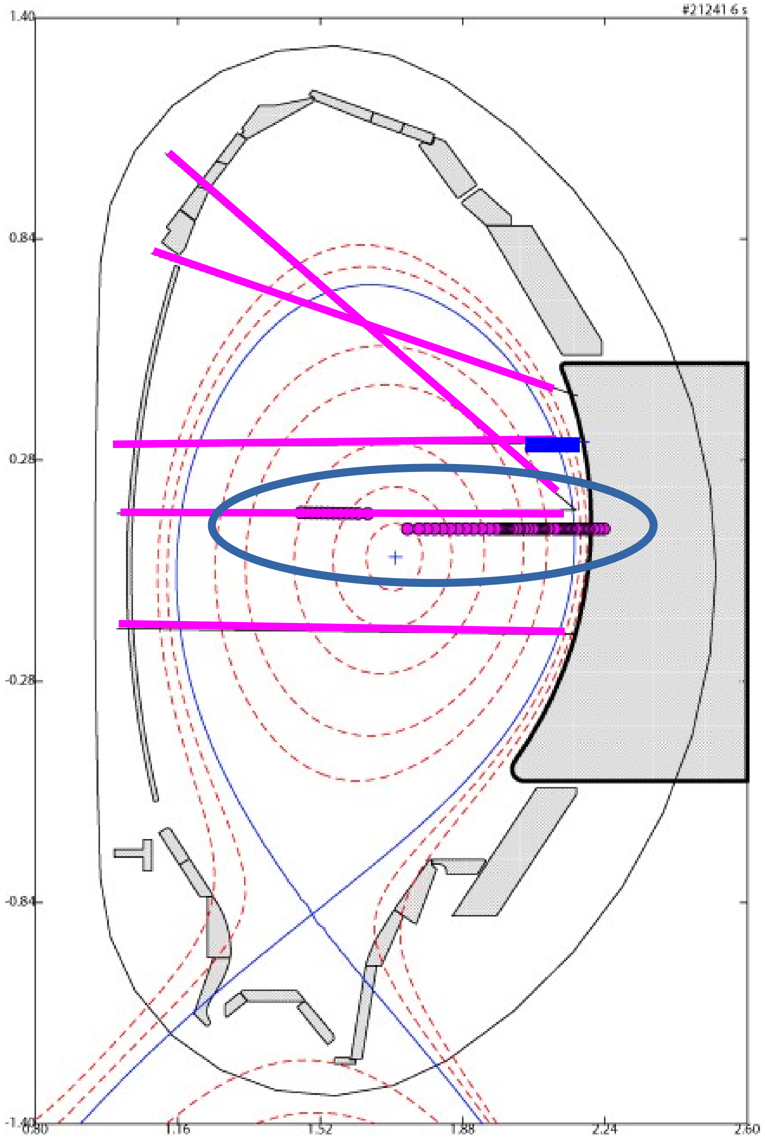
- temporal resolution 5 µs (5 ms with old LIB inversion technique)
- triggered installation of new optical head → fluctuation measurements

IDA: LIB + DCN + ECE

Electron cyclotron emission:

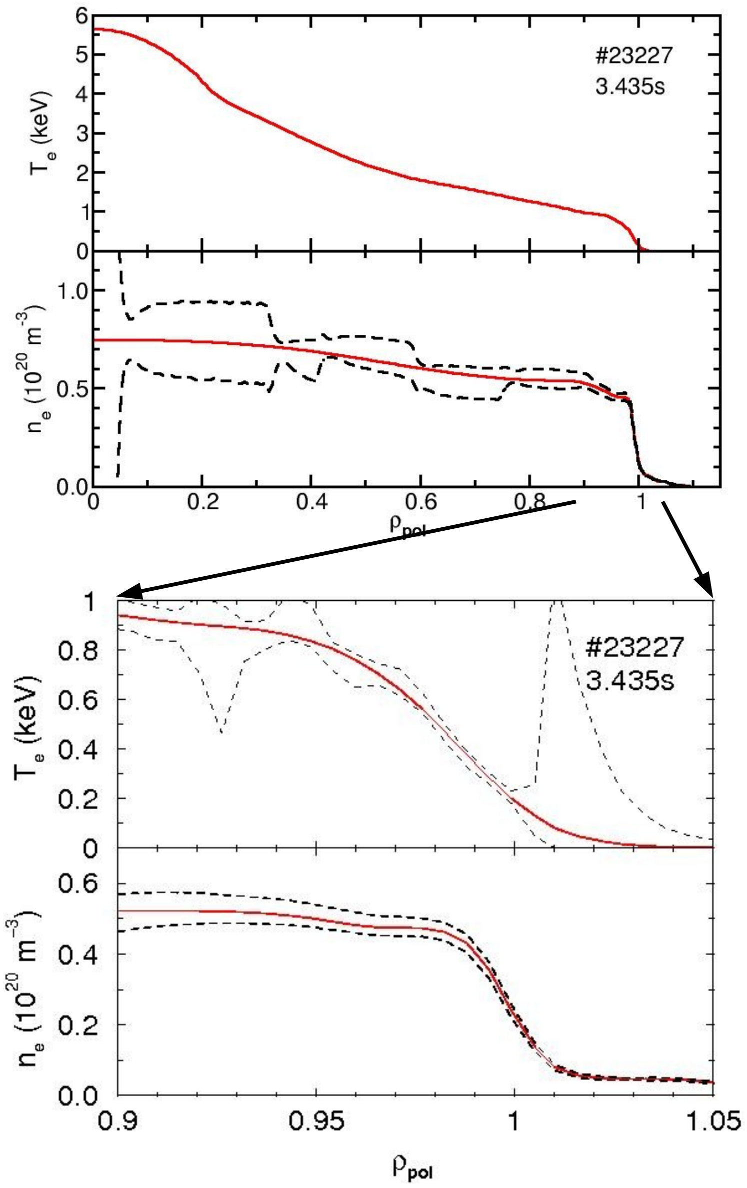
$$\omega_c = \frac{e\,B(R)}{m_e}$$



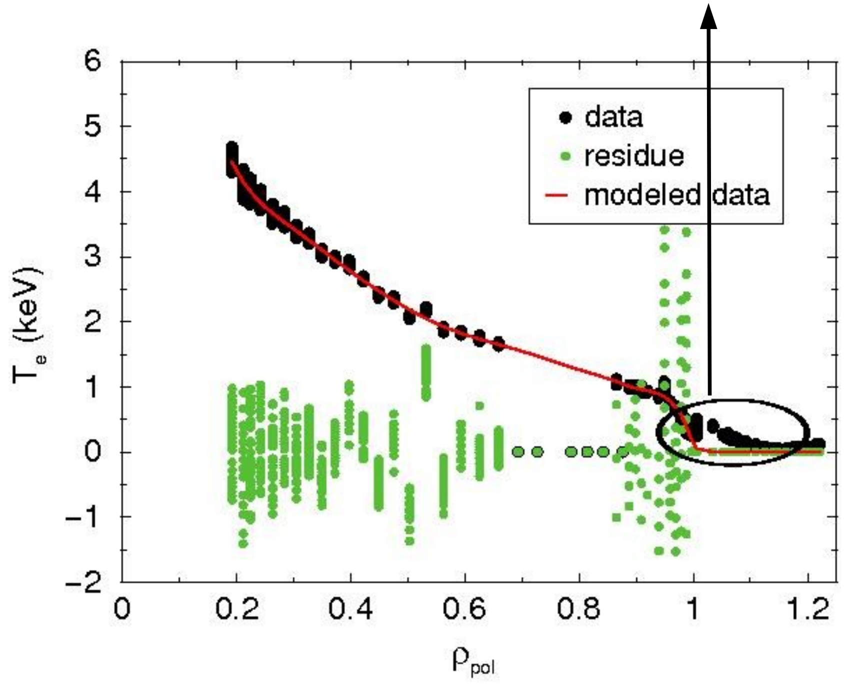


- optically thick plasma: blackbody radiation $d \propto T_e$
- optically thin plasma: broadened emission region $d \propto f\,(T_e, n_e)$

IDA: LIB + DCN + ECE



- ➔ simultaneous:
  - ✔ full density profiles
  - ✔ (partly) temperature profiles
  - ✔ pressure profile
- ➔ $n_e > n_{e,cut\text{-}off}$ → masking of ECE channels
- ➔ opt. depth ~ $n_e T_e$ → masking of ECE channels



IDA: LIB + DCN + ECE radiation transport

- Optically thick plasma: local emission and black-body radiation
- Optically thin plasma (edge and core)

  → EC emission depends on T_e and n_e

  → combination with data from density diagnostics is mandatory

  → calculate broadened EC emission and absorption profiles by solving the radiation transport equation

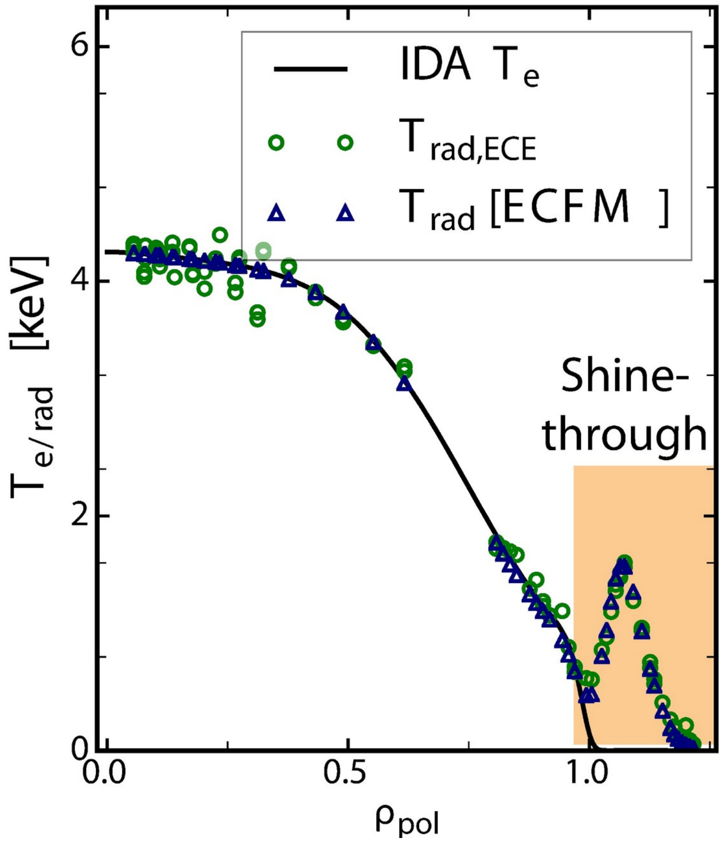
$$\frac{dI_\omega(s)}{ds} = j_\omega(s; n_e, T_e) - \alpha_\omega(s; n_e, T_e)\, I_\omega(s)$$

s	LOS coordinate
I_ω	spectral intensity
j_ω	emissivity
α_ω	reabsorption

→ electron cyclotron forward model (ECFM → ECRad) in the framework of Integrated Data Analysis

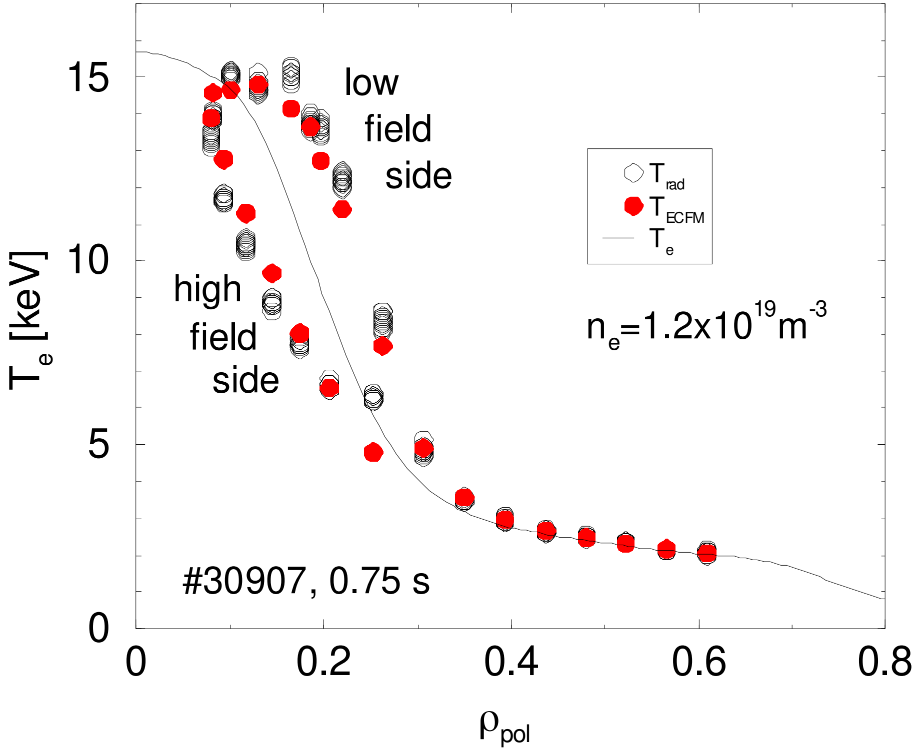
(S.K. Rathgeber et al., PPCF 55 (2013) 025004; S.S. Denk et al., PPCF 60 (2018) 105010)

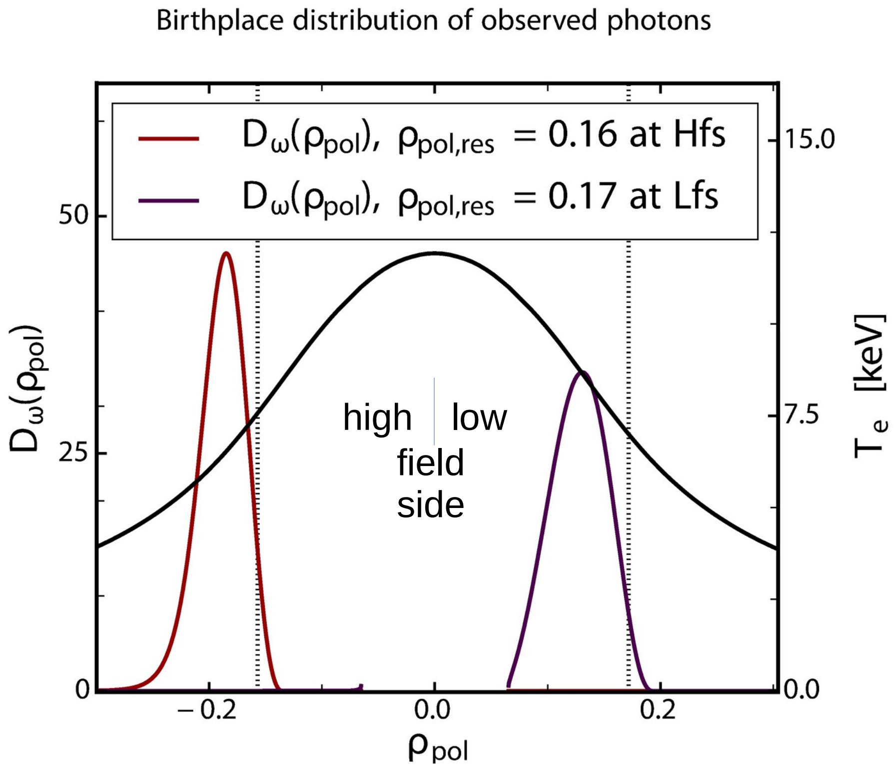
T_{Rad}, T_e for #30589 t = 1.27 s



edge shine-through

IDA: ECE radiation transport → Core Shine-through





ECE core hfs-lfs loop if small optical depth in core (small n_e) and large core T_e-gradient:

→ extended microwave emission region

→ high-field side: $T_{rad} < T_e$ due to shine-through of smaller (outer) temperatures

→ low-field side: $T_{rad} > T_e$ due to shine-through of larger (inner) temperatures

["Non-thermal electron distributions measured with ECE", S. Denk, Master thesis, 2014]

IDA: Uncertainties in Profiles (and Equilibria)

➢ How reliable are results from modeling codes (e.g. GENE, SOLPS, TGLF, ASTRA)?

uncertainty quantification (UQ), uncertainty propagation (UP), and *validation (V)*

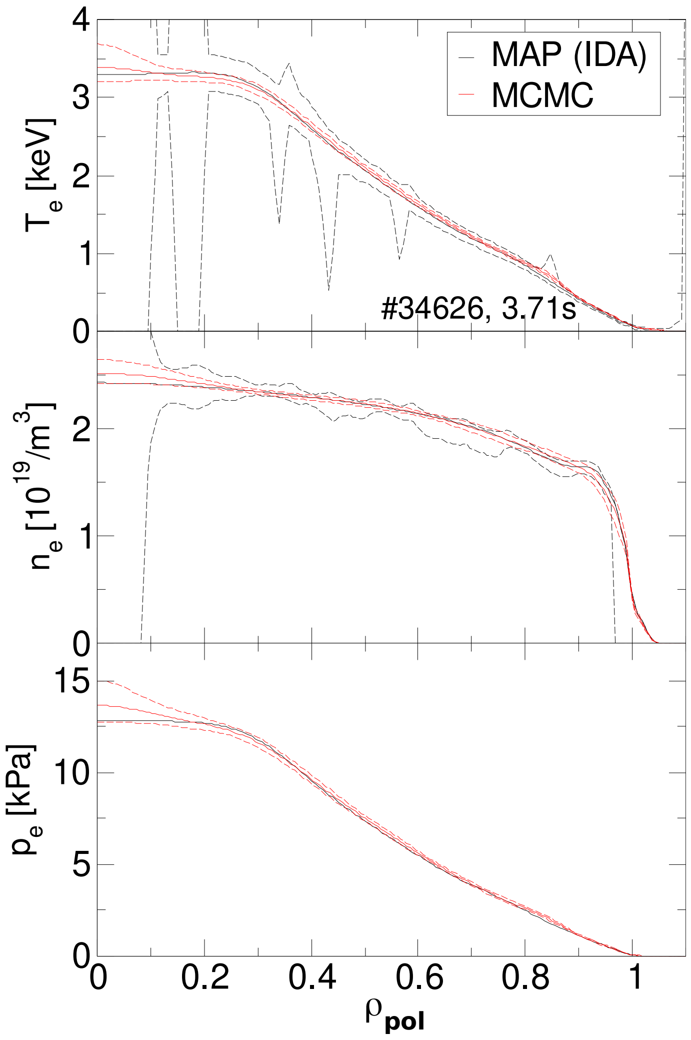
→ uncertain input quantities:

→ profiles (T_e, n_e, T_i, n_i, Z_{eff}, v_{rot}, n_{fast}, …) and uncertainties

→ profile gradients (dT_e/dR, $dT_e/d\rho$, $d\ln(T_e)/d\rho$, …) and uncertainties

→ equilibrium and uncertainties

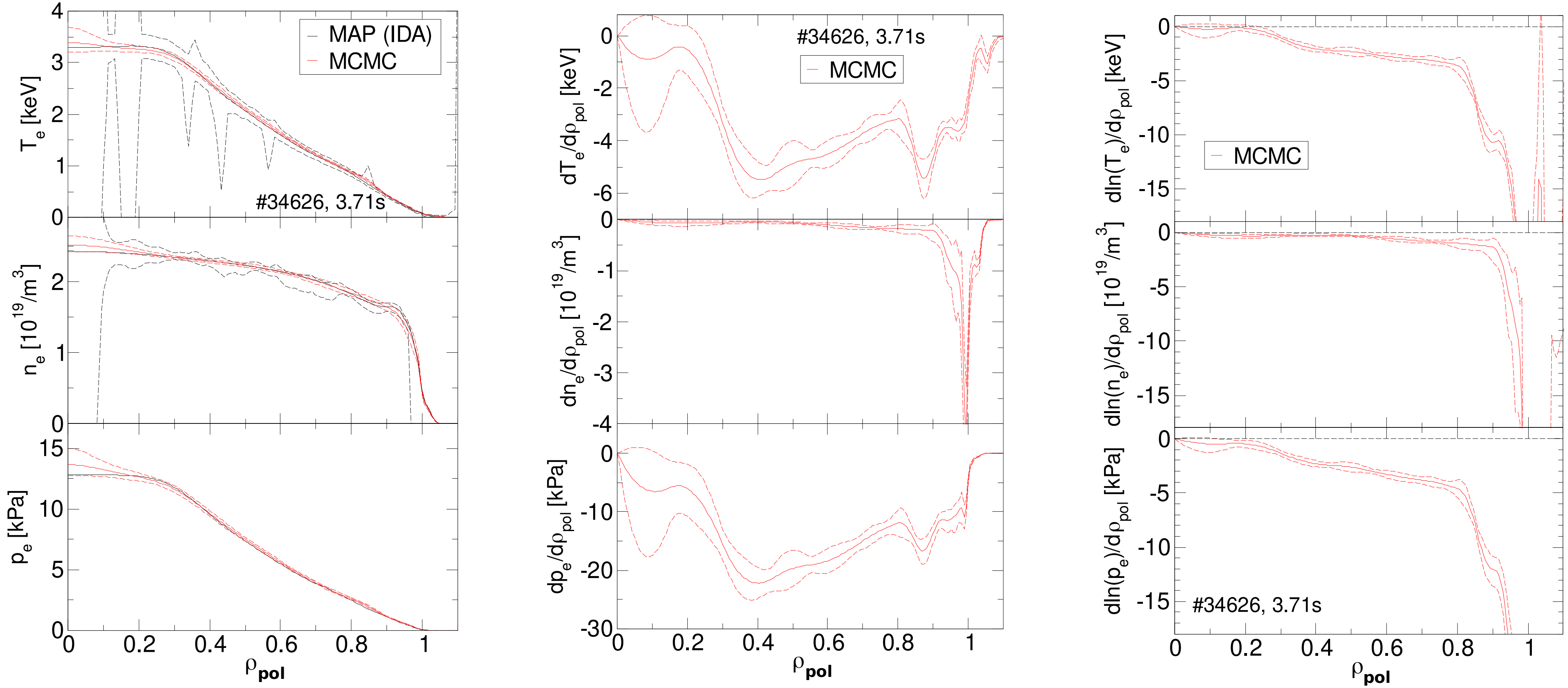
IDA: Uncertainties in Profiles: MAP, MCMC



- Maximum a Posteriori (MAP)
  → (T_e, n_e) estimate
  → error bar
    1. Covariance at probability maximum → local quantity
    2. from local profile changes and effect on χ^2 (Fischer, PPCF 2008) without profile correlations

- Markov chain Monte Carlo (MCMC) sampling of posterior pdf
  → mean → (T_e, n_e) estimate
  → variance → (small) error bar (incl. correlations)
  → profile samples
    → error propagation in modeling codes

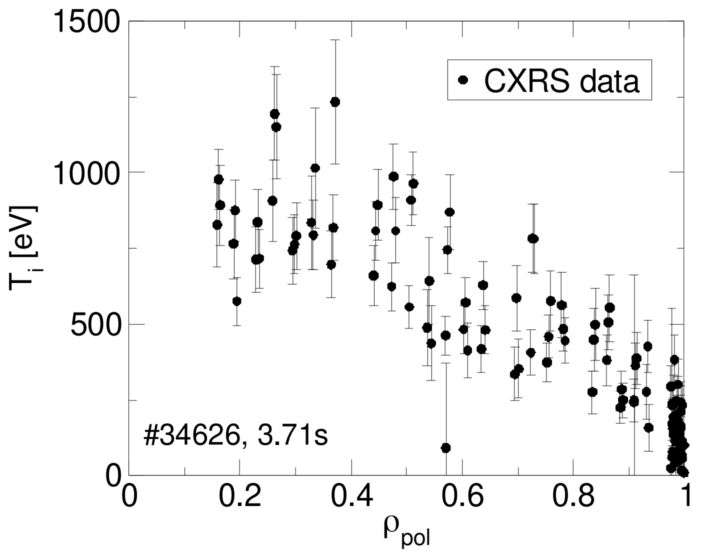
IDA: Logarithmic Gradients and Uncertainties



profiles | profile gradients | logarithmic profile gradients

→ GENE: UQ and UP (F. Jenko, C. Michoski, Univ. Texas Austin)

Gaussian Process Regression



R.M. McDermott, RSI 2017

- interpolation and smoothing of noisy data
- uncertainties of profiles and profile gradients
- extrapolation to axis

- Gaussian process (GP): random variable has (multidimensional) normal distribution
- GP regression (GPR):

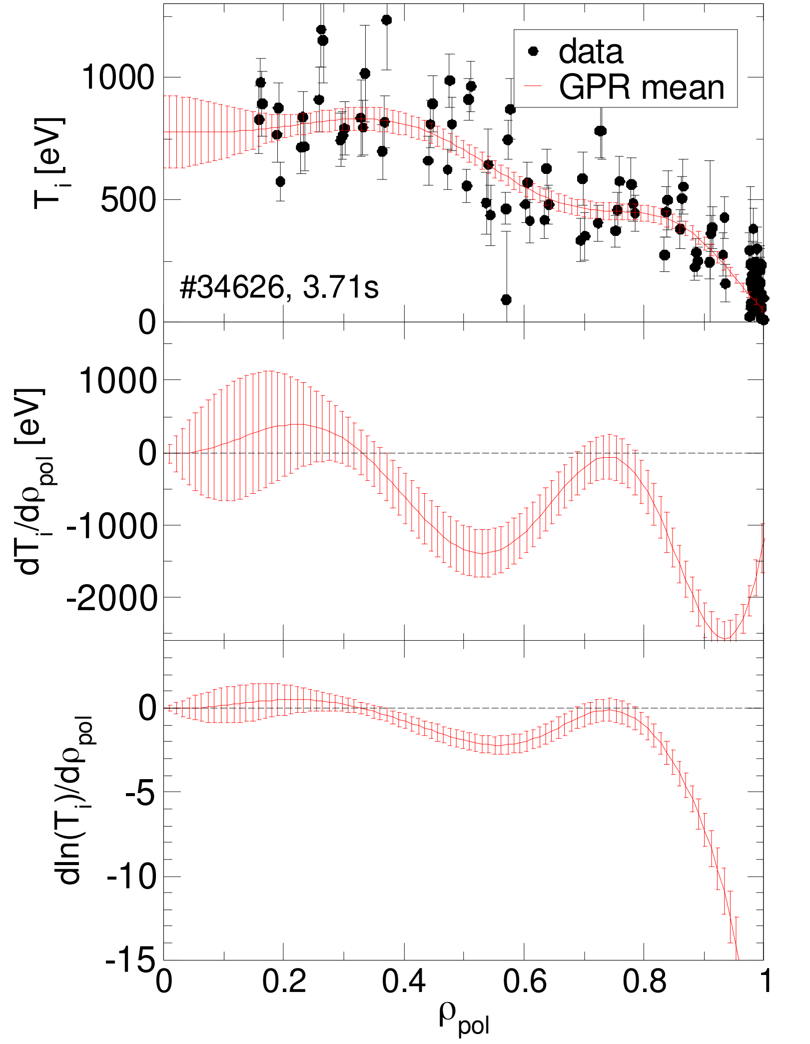
→ no assumption about profile shape!

→ different positions are correlated depending on distance $$Cov(f_k, f_l) = \eta^2 \exp\left(-\frac{(x_k - x_l)^2}{2\xi^2}\right)$$

→ likelihood $$p(\vec{d}\,|\,f(x), \vec{\sigma}) = N(\vec{d}\,|\,f(x), \vec{\sigma})$$

- Result: (Gaussian) probability distribution of possible interpolating functions
  - → Mean/variance of pdf: Analytic solution for profile, gradients and their uncertainties
  - → Samples of pdf: candidate profiles to study UP in modeling codes

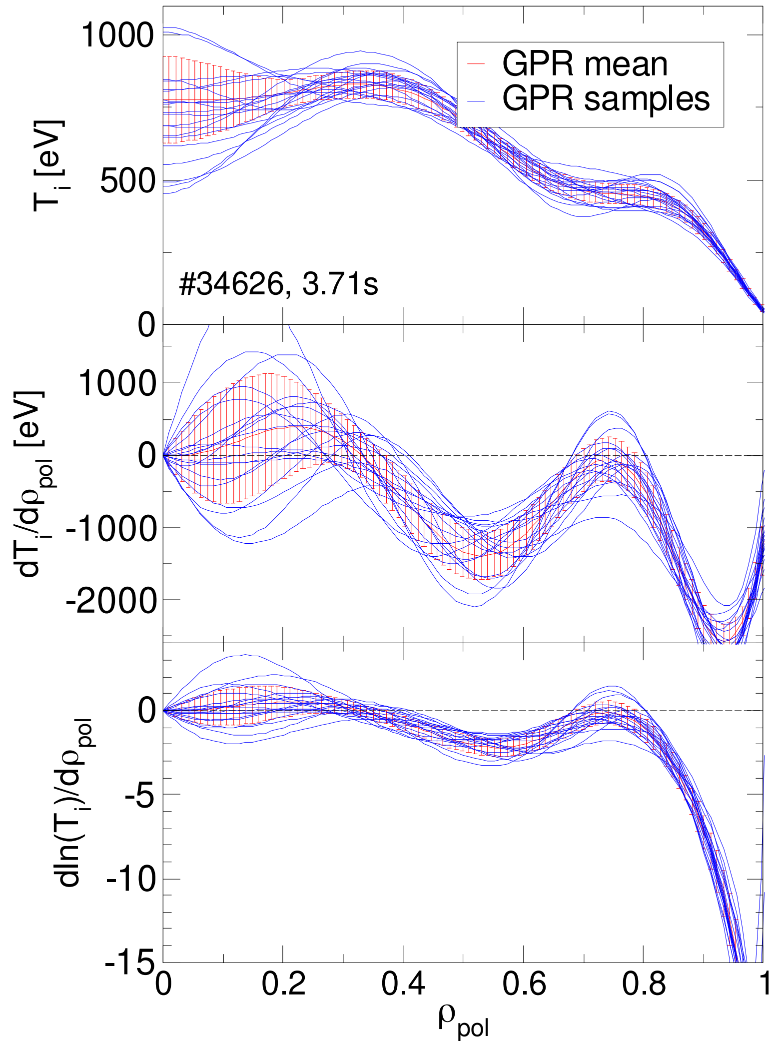
GPR: Profile, Gradient and Uncertainty



- estimation and uncertainty (1 std) of
  - → T_i profile
  - → T_i profile gradient
  - → T_i profile logarithmic gradient
- result depends on parameters:
  - → correlation length ξ (might depend on position ρ_{pol} : *non-stationary*)
    - ξ↓ → uncertainty↑
  - → kernel weight η
- constraint $dT_i/d\rho_{pol}=0$ at magn. axis

→ GENE: UQ and UP (F. Jenko, C. Michoski)

IDA: Samples of Profiles and Gradients



- samples of
  → T_i profile,
  → T_i profile gradient and
  → T_i profile logarithmic gradient

  useful for uncertainty propagation (UP)
  in modeling codes

- mean and uncertainty of
  profiles, gradients, logarithmic gradients
  and covariance matrices for sampling
  → fast: ~ 6s / 1000 time points
  → also for v_{tor}

IDA: Magnetic Equilibrium

The IDE equilibrium solver reconstructs the current distribution by solving the

1. Grad-Shafranov equation: Ideal magnetohydrodynamic equilibrium for poloidal flux function Ψ for axisymmetric geometry

$$\left(R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}+\frac{\partial^2}{\partial z^2}\right)\Psi=-(2\pi)^2\mu_0\left(R^2P'+\mu_0FF'\right)$$

subject to all available measured data (magnetics, pressure profile, polarimetry, (i)MSE, (SOL-)tile currents, loop voltages, iso-flux constraints, …)

and (non-physical) smoothness constraints to regularize the ill-conditioned solver

coupled with the

2. Current diffusion equation: describes the diffusion of the poloidal flux Ψ on the background of the toroidal flux $\Phi(\rho)$ due to resistivity

$$\sigma_{||}\frac{\partial\psi}{\partial t}=\frac{R_0J^2}{\mu_0\rho}\frac{\partial}{\partial\rho}\left(\frac{G_2}{J}\frac{\partial\psi}{\partial\rho}\right)-\frac{V'}{2\pi\rho}\left(j_{bs}+j_{ec}+j_{nb}\right)$$

Goal: replace non-physical smoothness constraints by a temporal correlation defined by the current diffusion

IDA: Magnetic Equilibrium: Sawtooth Crash

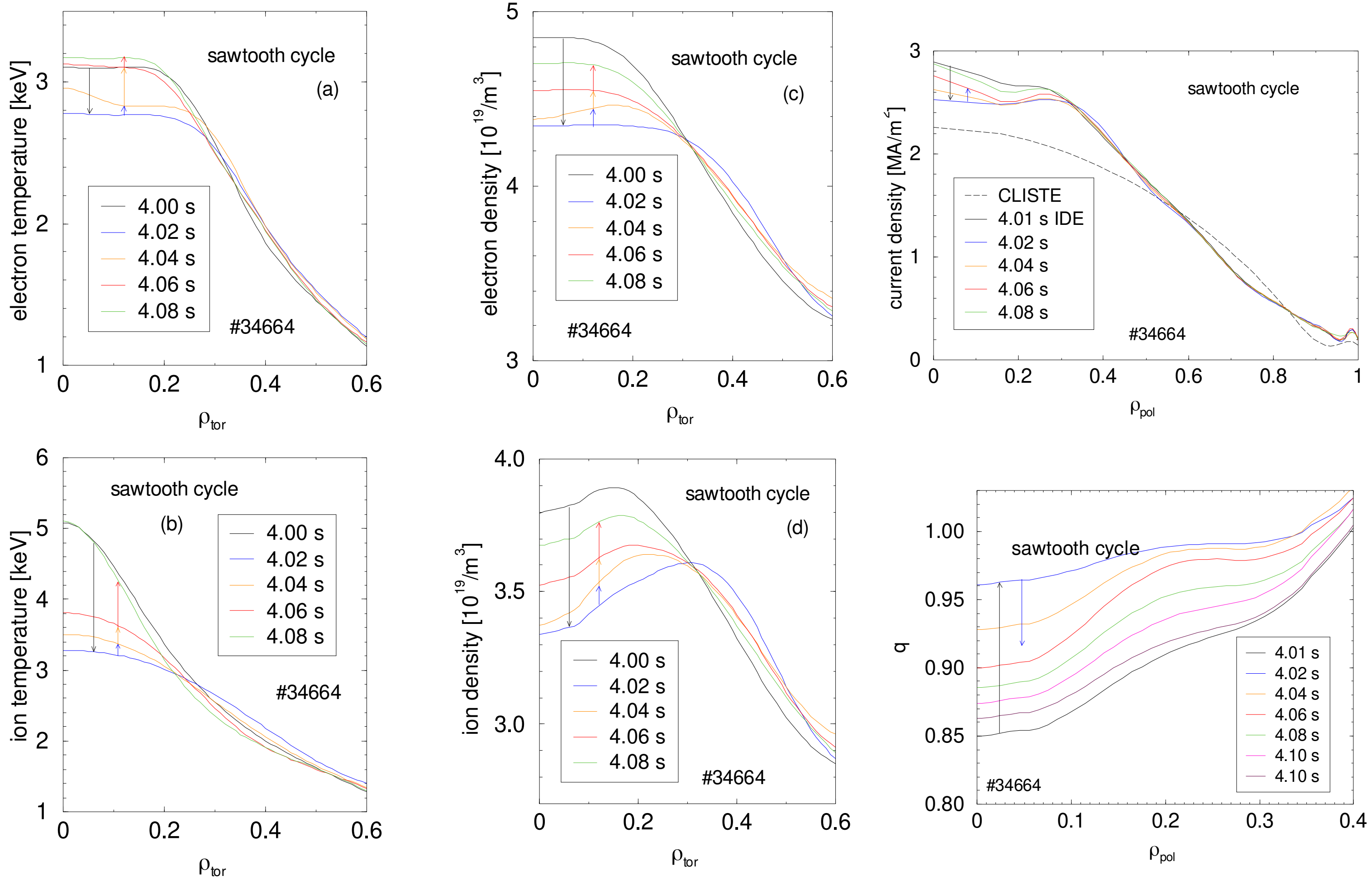
<u>Evolution of current distribution between sawtooth crashes:</u>

Most important ingredients:

1. GSE:
   + pressure profiles: $p_e + p_i + p_{fast}$
   (+ polarimetry)
   (+ MSE and iMSE)

2. CDE (neoclassical current diffusion):
   + kinetic profiles (→ conductivity → Te):
     T_e, n_e → IDA (central ECE!)
     T_i → GPR
     $n_i \sim f(n_e - n_{fast})$; Z_{eff} (bremsstrahlung)
   + j_{ECCD} from TORBEAM
   + j_{NBCD} from RABBIT (Weiland NF 2018)

3. Sawtooth times (soft X-ray)
   and current relaxation model
   (Kadomtsev or FCM (Fischer NF 2019))



Further Applications of IDA

W7-AS: n_e, T_e : TS, interferometry, soft X-ray

R. Fischer et al., PPCF, 45, 1095-1111 (2003)

ASDEX UG: n_e, T_e : TS, interferometry, ECE, …
Z_{eff} : bremsstrahlung spectra
T_i, v_{rot} : CXRS
equilibrium: Grad-Shafranov, current diffusion, many diagnostics

R. Fischer et al., FST, 58, 675-684 (2010)
S.K. Rathgeber et al., PPCF, 52, 095008 (2010)
R. Fischer et al., FST, 76, 879-893 (2020)
R. Fischer et al., NF, 59, 056010 (2019)

W7-X: non-Maxwellian electron energy distribution function: visible emission spectrum
n_e, $T_{e/i}$, impurity densities, flows: TS, X-ray imaging
n_e, T_e : TS, interferometry, helium beam
Z_{eff} : bremsstrahlung spectra

D. Dodt et al., J. Phys. D: Appl. Phys., 41:205207, 2008.
A. Langenberg et al., RSI, 90(6), 063505 (2019)
S. Kwak et al., arXiv:2103.07582, 2021
S. Kwak et al., RSI, 92:043505 (2021)

MST RFP: T_e : TS, soft X-ray
Z_{eff} : soft X-ray, CXRS

L. M. Reusch et al., RSI, 85:11D844, 2014.
M.E. Galante et al., NF, 55:123016, 2015.

TJ-II: n_e, T_e : TS, interferometry, reflectometry, Helium beam

B. Ph. van Milligen, et al., RSI 82, 073503 (2011)

JET: n_e : LIB
n_e, T_e : LIDAR, interferometry
fast-ion distributions : velocity-space tomography of fast-ion D-alpha spectroscopy, collective TS, gamma-ray and neutron emission spectrometry, and neutral particle analyzers.

D. Dodt, et al., P-2.148, EPS 2009
O. Ford, et al., P-2.150, EPS 2009
M. Salewski et al., FST 74:23–36, 2018.

R. Fischer et al., Integrated Data Analysis and Validation, Chap. 10, NF, to be published
arXiv:2411.09270 [physics.plasm-ph]

IDA Basic Implementation for ITER, JT-60SA, ...

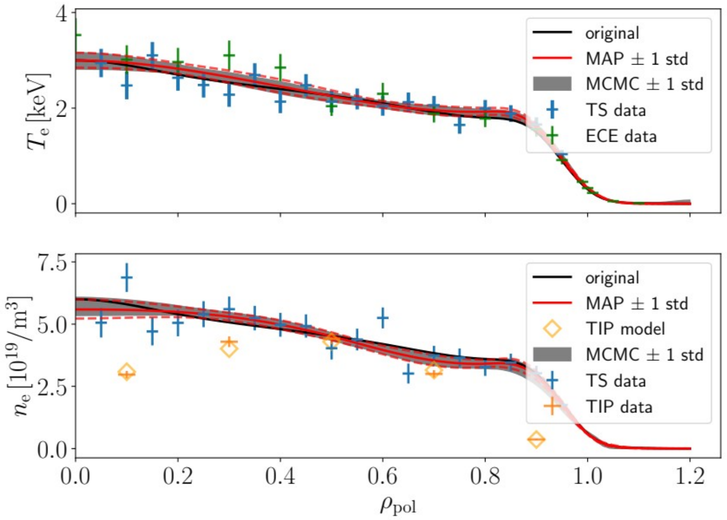
Basic implementation in python being completely modular

- to be compatible with any fusion device (ITER, DIII-D, JT-60SA, ...)
- diagnostics: Thomson scattering, ECE and interferometry, ...
- likelihoods (data uncertainty): Gaussian, Cauchy (outlier robust), …
- multi-fidelity forward models / synthetic diagnostics
  - ECE: $T_{rad} = T_e$ vs radiation transport modeling $T_{rad}(T_e, n_e)$
  - real-time vs offline analysis
- flexible parameterisation of, e.g., profiles: splines, GPR, ...
- priors: smoothness, positivity, physical modeling, ...
- results and their uncertainties:
  - MAP solution (probability maximum and width)
  - MCMC sampling methods (explore full probability space)

```
.
├── run_ida_sample.py
└── lib
    ├── core
    │   ├── likelihoods
    │   ├── optimizers
    │   └── IDA
    └── diagnostics
        ├── base
        ├── AUG
        │   ├── ECE
        │   ├── INTF
        │   └── TS
        ├── DIII-D
        ├── ITER
        └── JT-60SA
```

IDA: ITER workflow

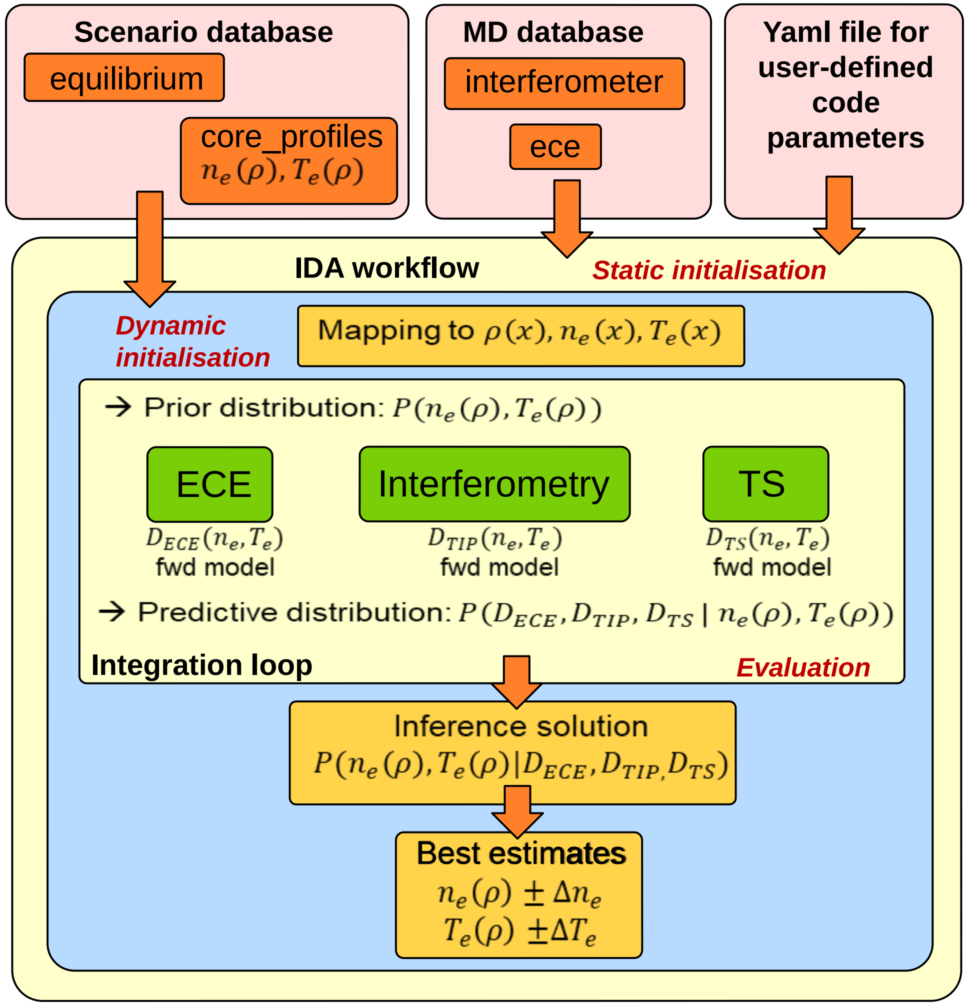
- artificial diagnostics: Thomson scattering, ECE
  - → synthetic data set with 10% noise
- 1st ITER diagnostic: Toroidal Interferometer Polarimeter (TIP)
  - → synthetic data set with 5% noise
  - → IMAS synthetic diag.



- MAP ±1std
- MCMC (50±34)% percentile

IMAS Interface Data Structures (IDS):

- read: TIP geometry (interferometer_md), equilibrium
- write: results ...



M. Schneider

IDA in the Bayesian framework for Nuclear Fusion

Bring together different diagnostics/diagnosticians/theoreticians with redundant/complementary/modeling data

- Probabilistic modeling of individual diagnostics (forward models, likelihoods for all kind of uncertainties)
- Probabilistic combination of different diagnostics (multiply pdfs, unified error analysis, error propagation)
- Probabilistic combination with prior / modeling information

- Redundant data:
  - ➢ more reliable results by larger (meta-) data set → reduction of estimation uncertainties
  - ➢ detect and resolve data inconsistencies (reliable/consistent diagnostics) using standardized error/uncertainty treatment

- Complementary data:
  - ➢ resolve parametric entanglement
  - ➢ resolve complex error propagation (non-Gaussian)
  - ➢ synergistic effects (exploiting full probabilistic correlation)
  - ➢ automatic *in-situ* and *in-vivo* calibration (transient effects, degradation, …)
  - ➢ advanced data analysis technique → improvements in modeling (ECE) and diagnostics hardware (LIB)

- Goal: Coherent combination of measurements from different diagnostics
  - ➢ replace combination of results from individual diagnostics
  - ➢ with combination of measured data → one-step analysis of pooled data
  - ➢ in a probabilistic framework

Science of Logic

The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind.

James Clerk Maxwell (1850)