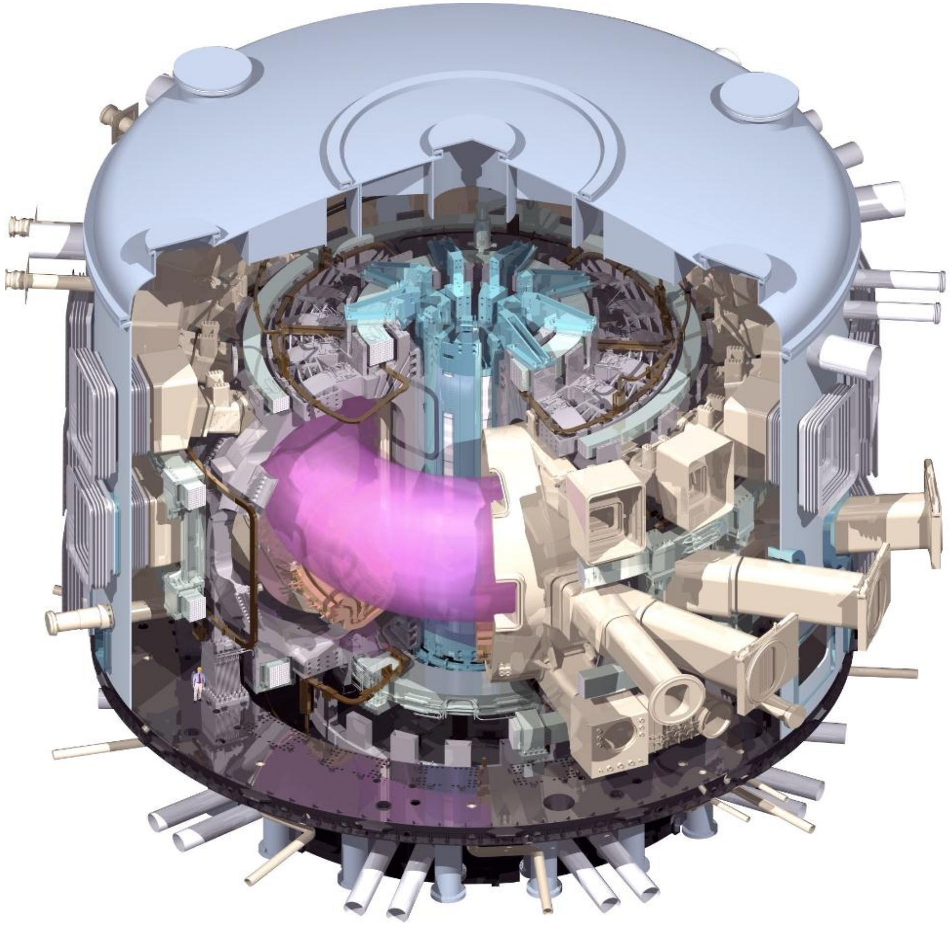
Nonlinear Modelling of Fast Ion Driven Instabilities in Fusion Plasmas

SD Pinches
ITER Organization



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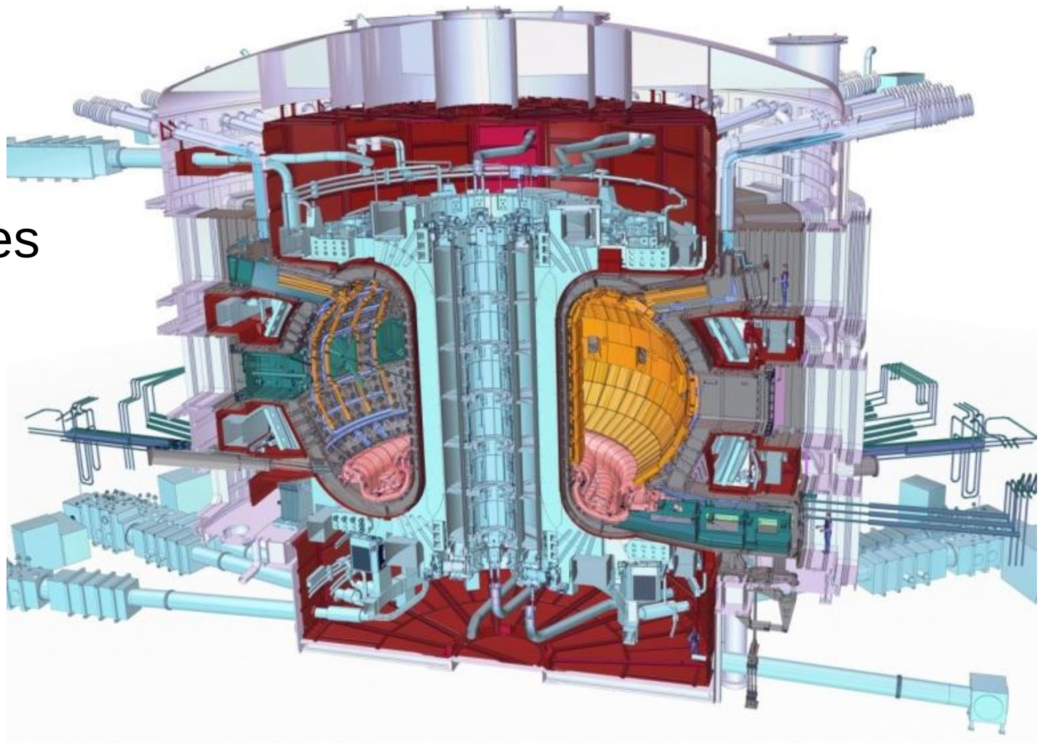
Outline of Talk

- Introduction to fast ions and fast ion driven modes
- Overview of the HAGIS code
- Nonlinear modelling of fast ion driven instabilities
  - Growth and saturation
  - Multiple modes interacting
  - Pitchfork splitting
  - Frequency sweeping modes
  - Fishbones
  - Tornado modes
- Summary



ITER Mission

- The overall programmatic objective:
  - to demonstrate the scientific and technological feasibility of fusion energy for peaceful purposes

- The principal goal:
  - to design, construct and operate a tokamak experiment at a scale which satisfies this objective

- **ITER is designed to confine a Deuterium-Tritium plasma in which α-particle heating dominates all other forms of plasma heating:**

$\Rightarrow$ a burning plasma experiment

ITER Mission

Physics:

- Produce a significant fusion power amplification factor (Q ≥ 10) in long-pulse operation (300 – 500 s)
- Aim to achieve steady-state operation of a tokamak (Q ≥ 5, ≤ 3000 s)
- Retain the possibility of exploring 'controlled ignition' (Q ≥ 30)

Technology:

- Demonstrate integrated operation of technologies for a fusion power plant
- Test components required for a fusion power plant
- Test concepts for a tritium breeding module

Burning plasma physics in ITER

- **Access to plasmas which are dominated by α-particle heating will open up new areas of fusion physics research, in particular:**
  - confinement of α-particles in plasma
  - response of plasma to α-heating
  - influence of α-particles on stability

- **Experiments in existing tokamaks have already provided some positive evidence**
  - 'energetic particles' (including α-particles) are well confined in the plasma
  - 'energetic particle' populations interact with the background plasma and transfer their energy as predicted by theory
  - but 'energetic particles' can drive instabilities (Alfvén eigenmodes) - for ITER parameters at Q=10, the impact is predicted to be tolerable

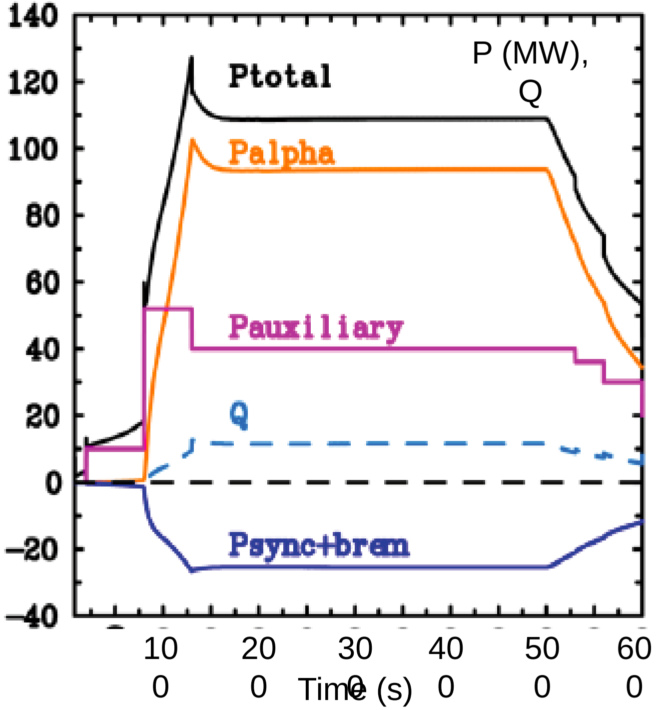
ITER Baseline Reference Scenarios

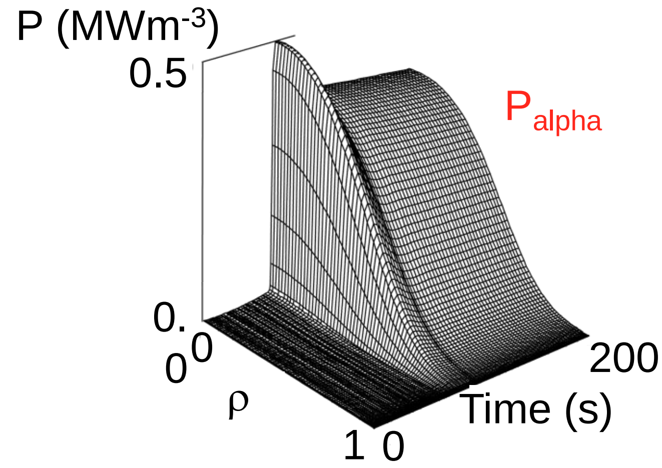
- The set of DT reference scenarios in ITER is specified via illustrative cases in the *Project Requirements* $\Rightarrow$ Design Basis scenarios

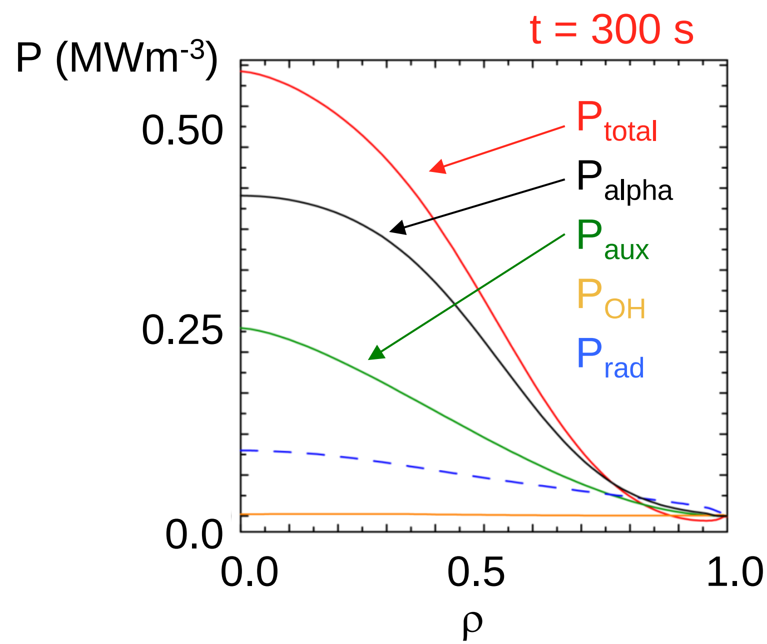
Parameter	Inductive Operation	Hybrid Operation	Non-inductive Operation
Plasma Current, I_p (MA)	15	13.8	9
Safety Factor, q_{95}	3.0	3.3	5.3
Confinement Time, τ_E (s)	3.4	2.7	3.1
Fusion Power, P_{fus} (MW)	500	400	360
Power Multiplication, Q	10	5.4	6
Burn time (s)	300 – 500	1000	3000

In addition, a range of non-active (H, He) and D plasma scenarios must be supported for commissioning purposes to support rapid transition to DT operation

Alpha-particle heating at Q = 10







- **As the alpha power rises in high-Q plasmas, the plasma will enter a novel regime**
  - Plasma behaviour dominated by α-particle heating

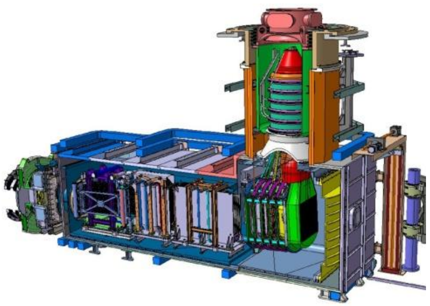
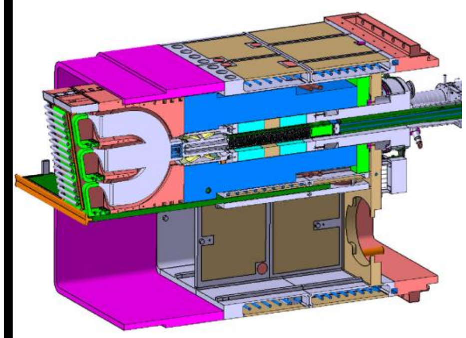
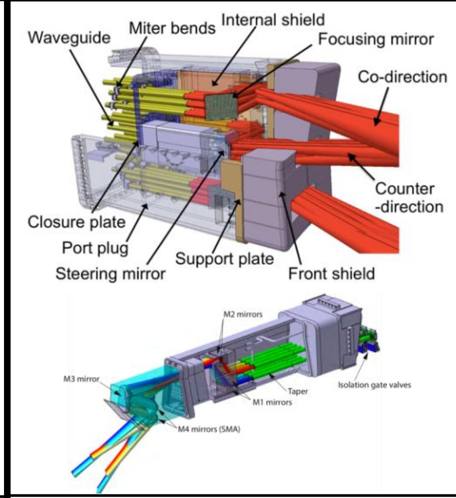
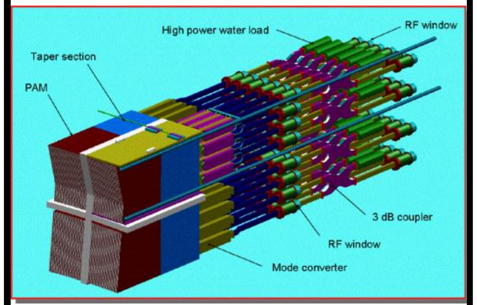
⇒ Burning plasma regime

SH Kim

Sources of Energetic Particles

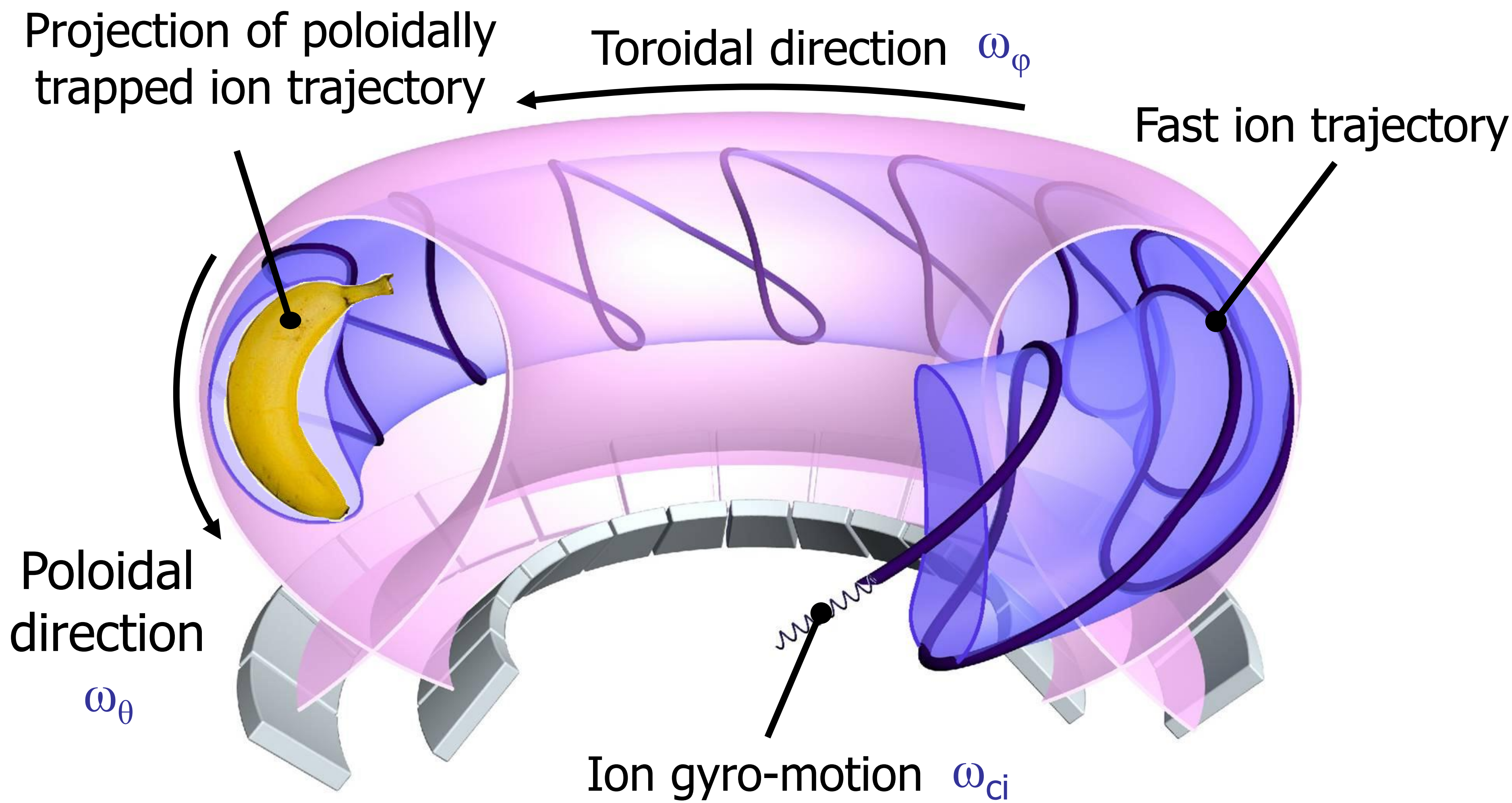
- Nuclear fusion
  - Isotropic slowing-down distribution
  - For DT fusion, α-particle birth energy of 3.5 MeV
- Neutral beam injection (NBI)
  - Anisotropic slowing-down distribution
  - Well defined E_b
- Radio Frequency (RF)
  - E.g. Ion Cyclotron (ICRH)
  - No well defined characteristic energy
  - Anisotropic

ITER Heating and Current Drive Systems

NB	IC	EC	LH
Neutral Beam -1 MeV	Ion Cyclotron 40 – 55 MHz	Electron Cyclotron 170 GHz	Lower Hybrid ~5 GHz
			
33MW* +16.5MW#	20MW* +20MW#	20MW* +20MW#	0MW* +40MW#
Bulk current drive limited modulation	Sawtooth control modulation < 1 kHz	NTM/sawtooth control modulation up to 5 kHz	Off-axis bulk current drive

*Baseline Power #Possible Upgrade

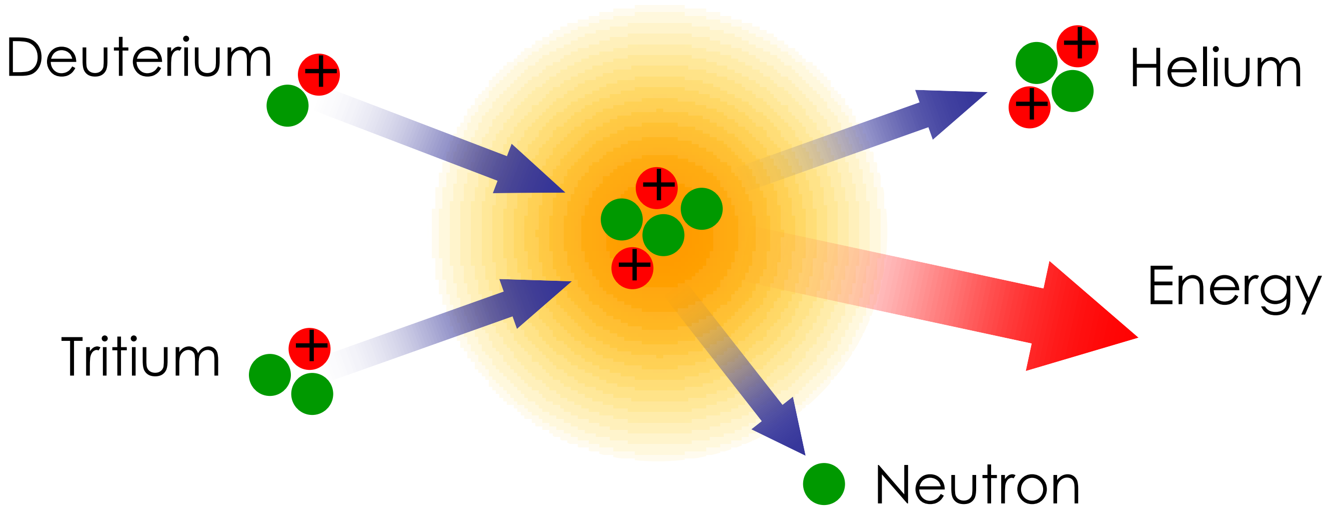
Fast Ion Orbits



Various natural frequencies associated with particle motion

Burning Plasmas

- New physics element in burning plasmas:
  - Plasma is self-heated by fusion alpha particles



$v_{Ti} \ll v_A < v_\alpha \ll v_{Te}$

ITER parameters

$v_{Ti} = 0.9\times10^6$ m/s

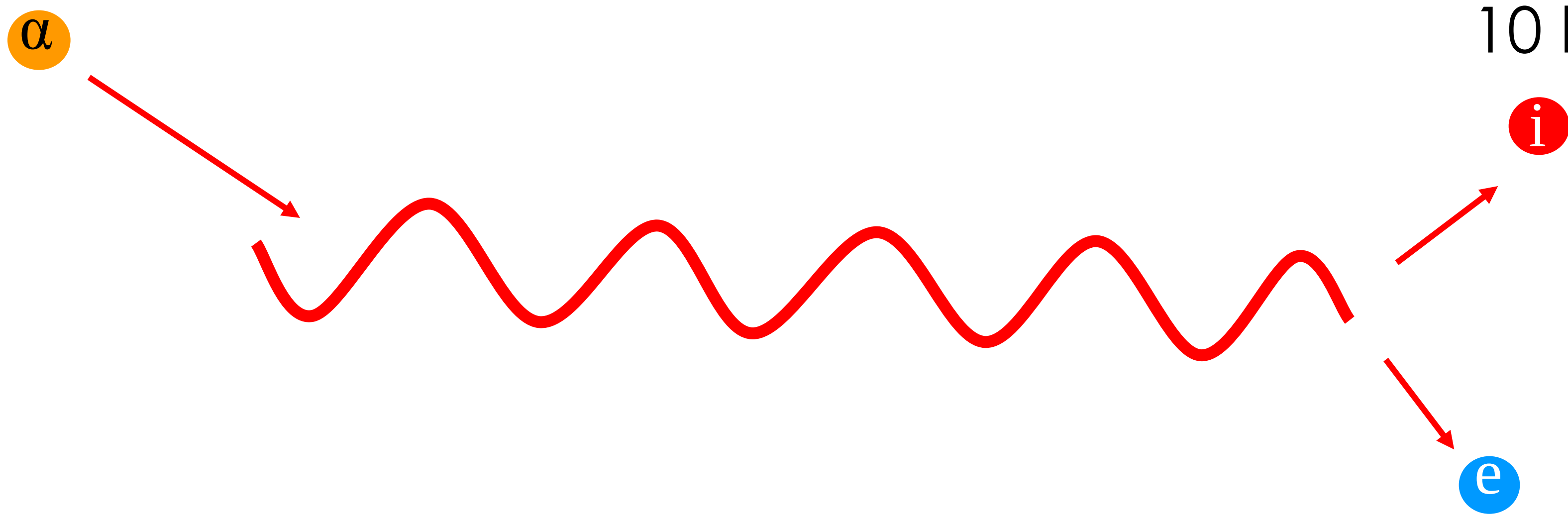
$v_A = 8\times10^6$ m/s

$v_\alpha = 12\times10^6$ m/s

$v_{Te} = 59\times10^6$ m/s

Alfvén waves and αs

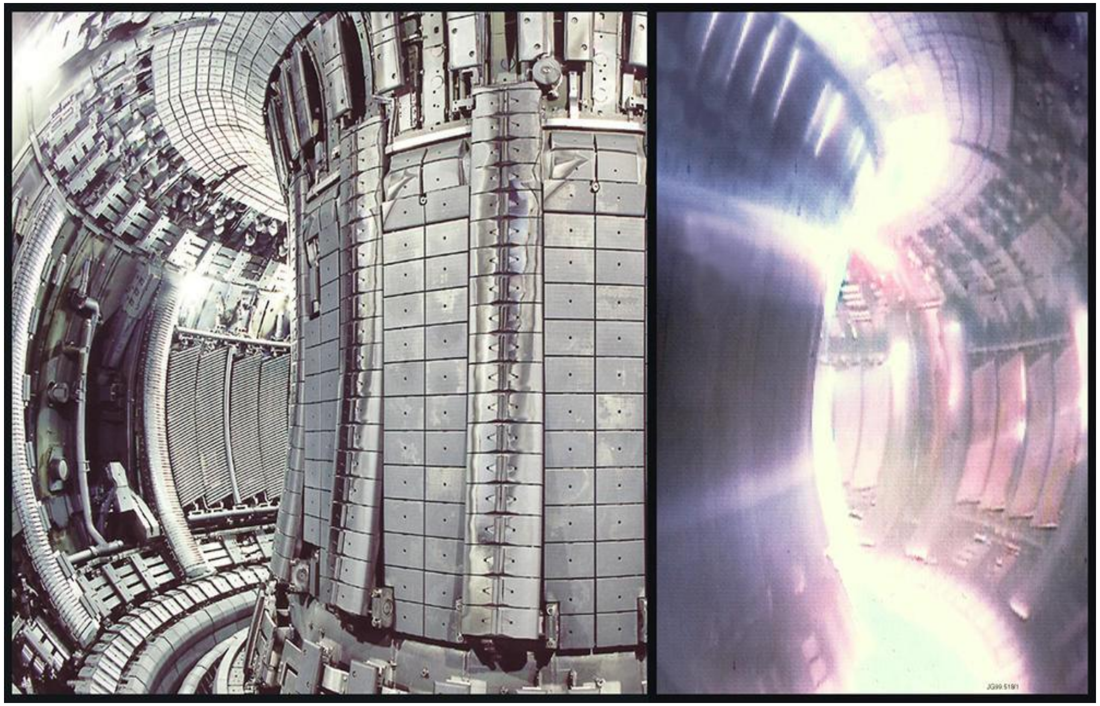
Alfvén wave is *very* weakly damped by background plasma



Fusion products (αs) interact with Alfvén waves *much* better than thermal plasma

Loss of Fast Particles

- Loss of bulk plasma heating
  - Clearly unacceptable for an efficient power plant
- Damage to first wall
  - Can only tolerate losses of a few % in a reactor

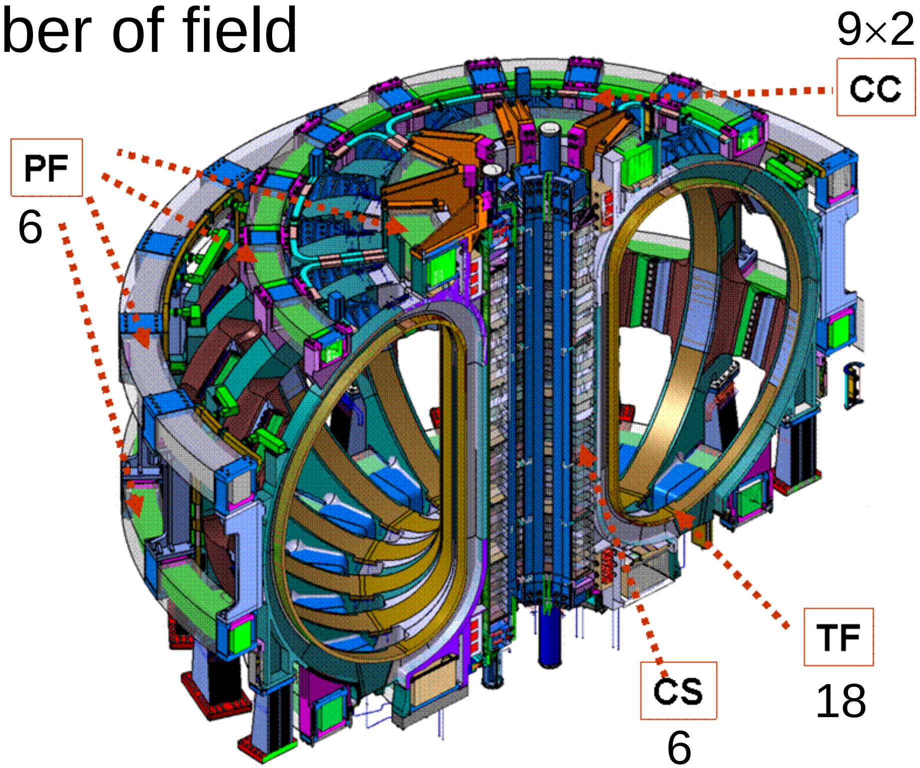


Reasons for Loss

- Imperfections in confining magnetic field
  - Ripple due to finite number of field coils, TBMs, ELM coils

48 superconducting coils

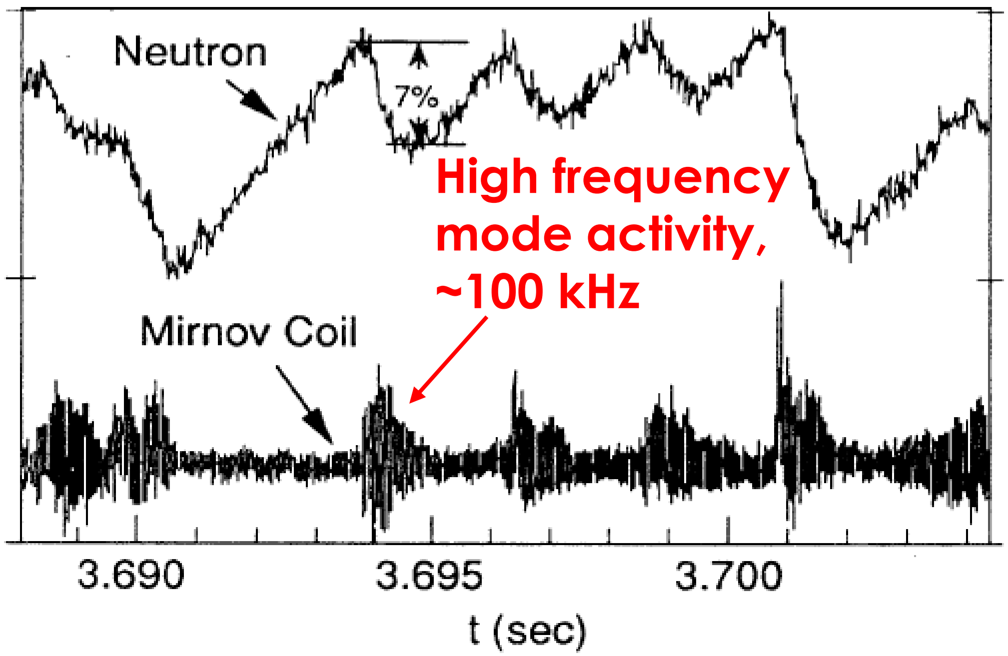
System	Energy GJ	Peak Field	Total MAT	Cond length km	Total weight t
Toroidal Field TF	41	11.8	164	82.2	6540
Central Solenoid	6.4	13.0	147	35.6	974
Poloidal Field PF	4	6.0	58.2	61.4	2163
Correction Coils CC	-	4.2	3.6	8.2	85



- Self-generated field imperfections
  - Collective instabilities

Wave Induced Losses in TFTR

- Specially designed experiments
  - Low field, $B_t = 1$ T
  - Deuterium NBI, $E_b({}_0D^2) = 100$ keV
  - $v_b \sim v_A$
- Modes observed for $P_{NBI} > 5$ MW

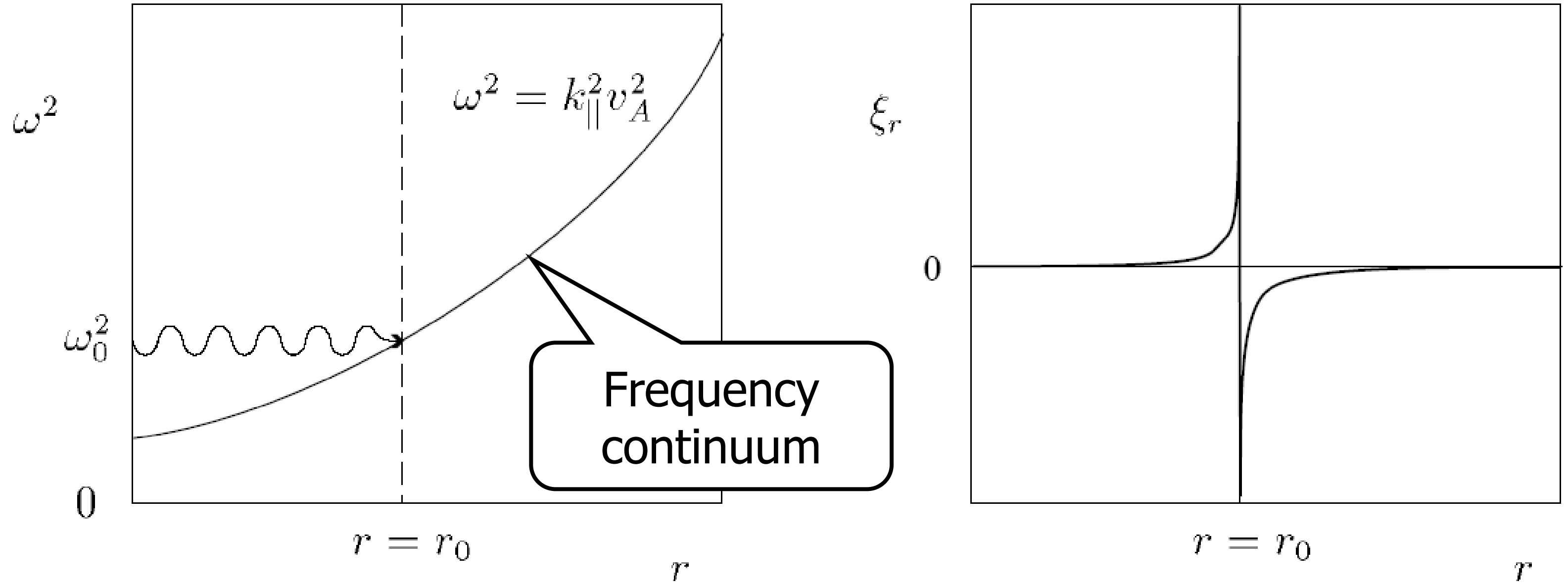


- Correlated with neutron reduction
  - Neutron yield dominated by beam-plasma reactions
    $\Rightarrow$ ***Fast ion loss***

K.L. Wong *et al*., Phys. Rev. Lett. **66** (1991)

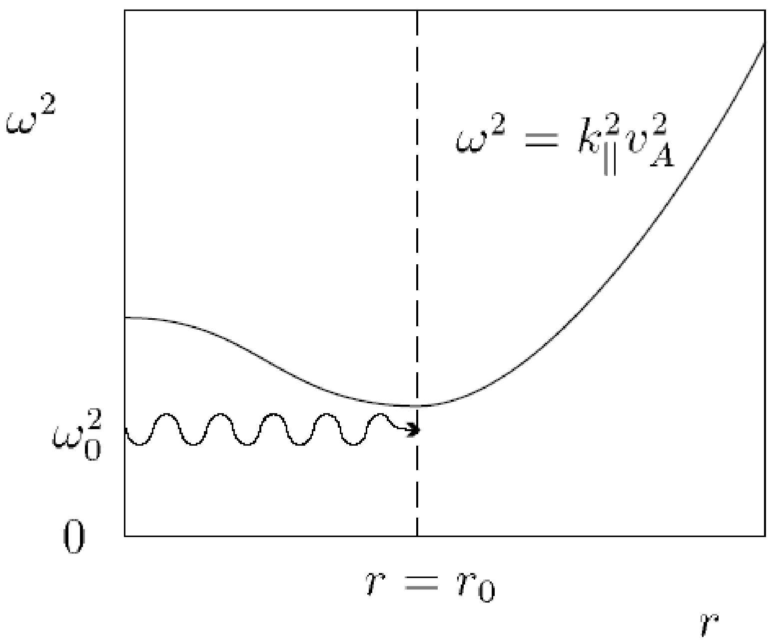
Alfvén Waves

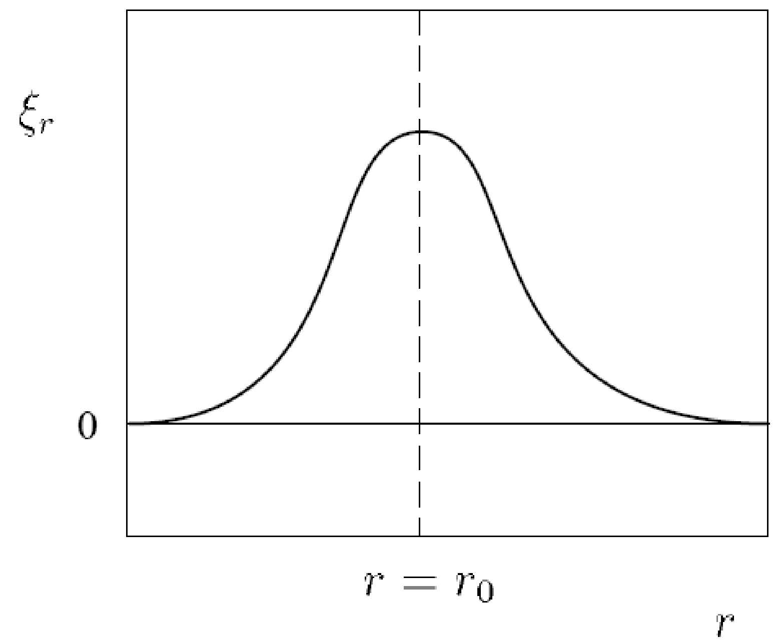
- Analogous to waves on a string
  - $v_A = B/\sqrt{(\mu_0 m_i n_i)}$
  - $\omega^2 = \omega_A^2(r) \equiv k_\parallel^2 v_A^2(r)$
  - Form continuum of waves in inhomogeneous plasma
  - Damped due to phase mixing with neighbouring waves



Alfvén Waves and Eigenmodes

- Current carrying inhomogeneous cylinder:
  - Helical field
  - $k_{||} = k_{||}(r)$
  - Continuum has extremum
  - Global Alfvén Eigenmode (GAE)

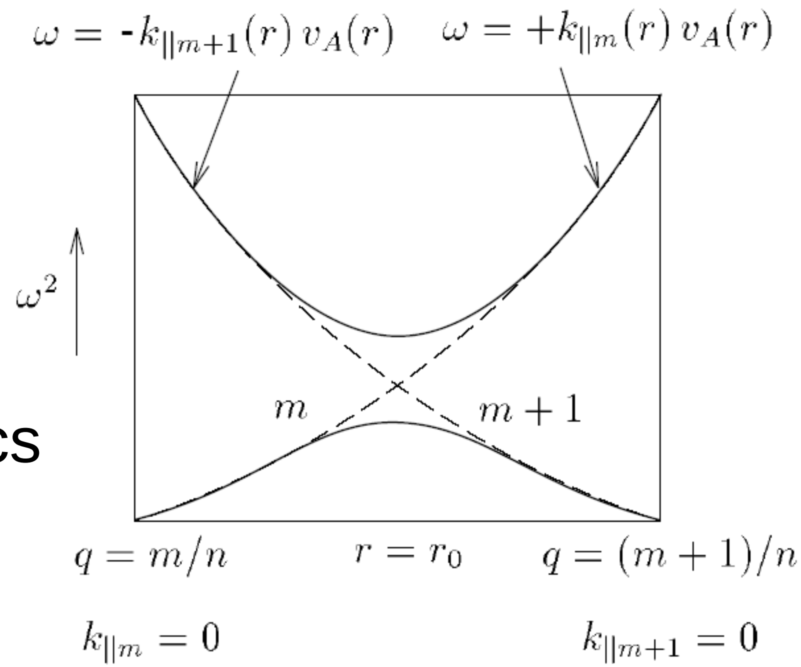


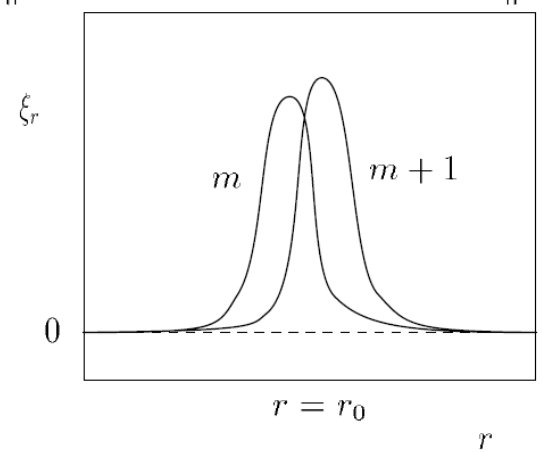


K. Appert *et al.*, Plasma Phys. **24** (1982), D. W. Ross *et al.*, Phys. Fluids **25** (1982)

Alfvén Waves in Tori

- Tokamak plasma:
  - Fourier decomposition:
    - A ~ exp[i(nϕ - mθ - ωt)]
  - $B \approx B_0 R_0 / R \approx B_0 \left(1 - r/R_0 \cos\theta\right)$
  - Neighbouring poloidal harmonics couple due to toroidicity
  - Gaps in frequency continuum
  - Toroidal Alfvén Eigenmodes (TAE) exist in frequency gap
    - Weakly damped
  - $f_{TAE} \sim v_A/(2qR)$





C. Z. Cheng, Liu Chen and M. S. Chance, Ann. Phys. **161** (1985)

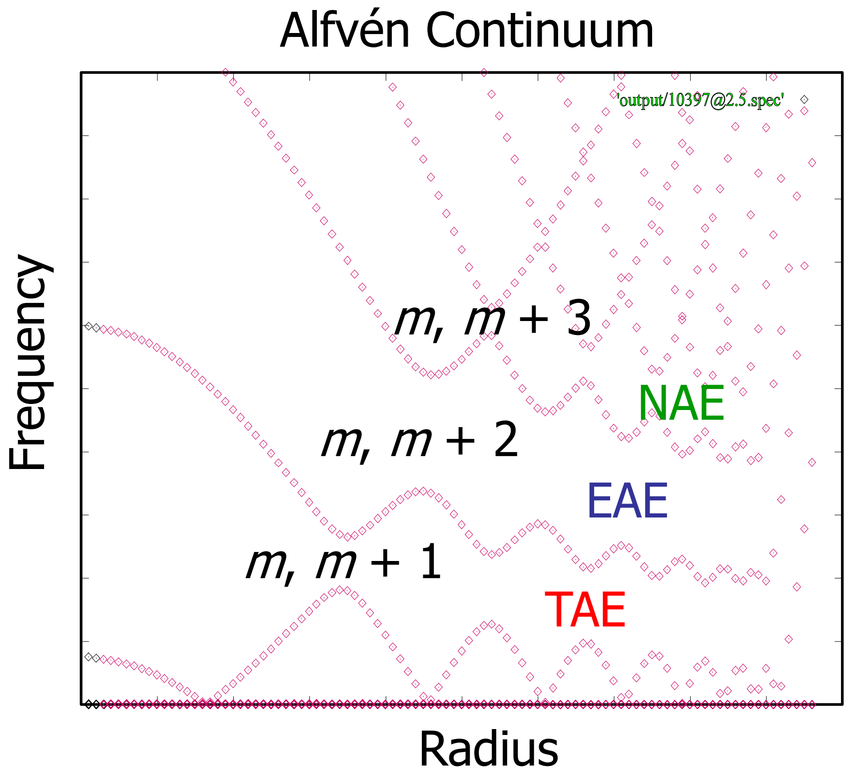
Alfvén Eigenmodes

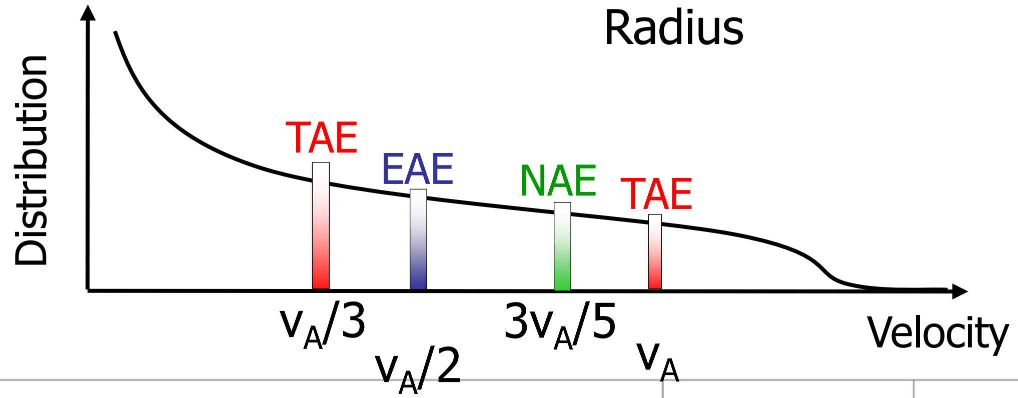
- Exist in frequency gaps
- Comprise of two primary harmonics, *m* and *m* + *L*
  - **Wave-particle resonance condition:**

    $\omega - n\,\omega_\phi + (m \pm 1)\,\omega_\theta = 0$

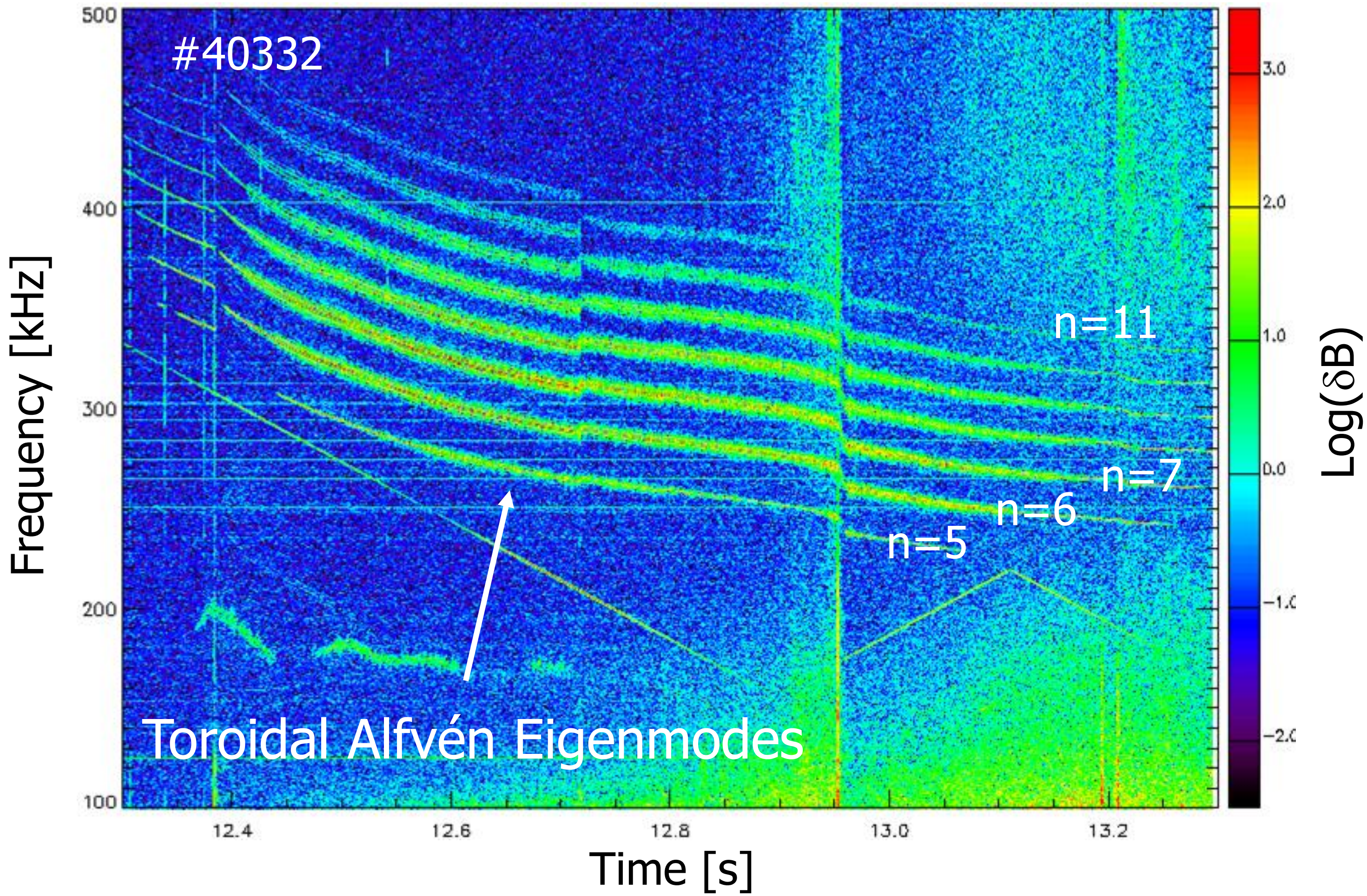
$$\mathbf{v_{||} = \pm L/(2 \pm L)\, v_A}$$

  - TAE: L = 1
  - EAE: L = 2
  - NAE: L = 3





TAE in JET driven by ICRH accelerated ions

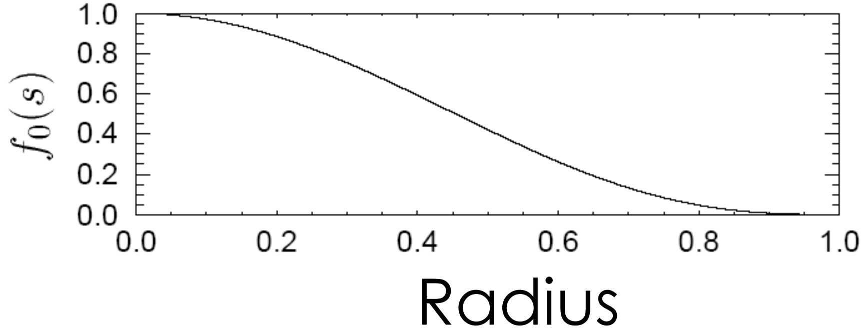


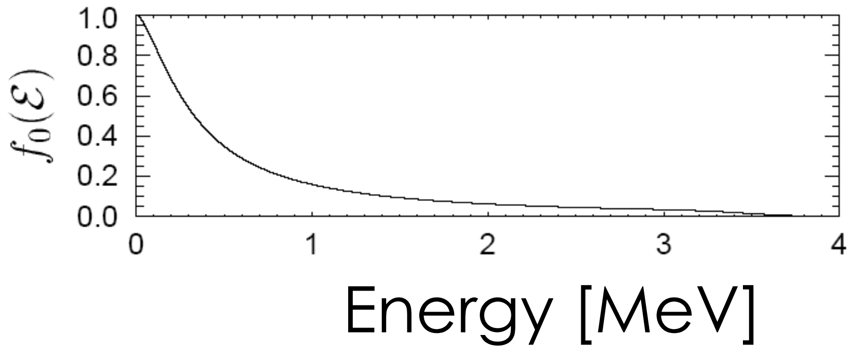
- TAE have constant amplitude and fine frequency splitting

  $\Rightarrow$ **Nonlinear effect**

Fast Particle Drive

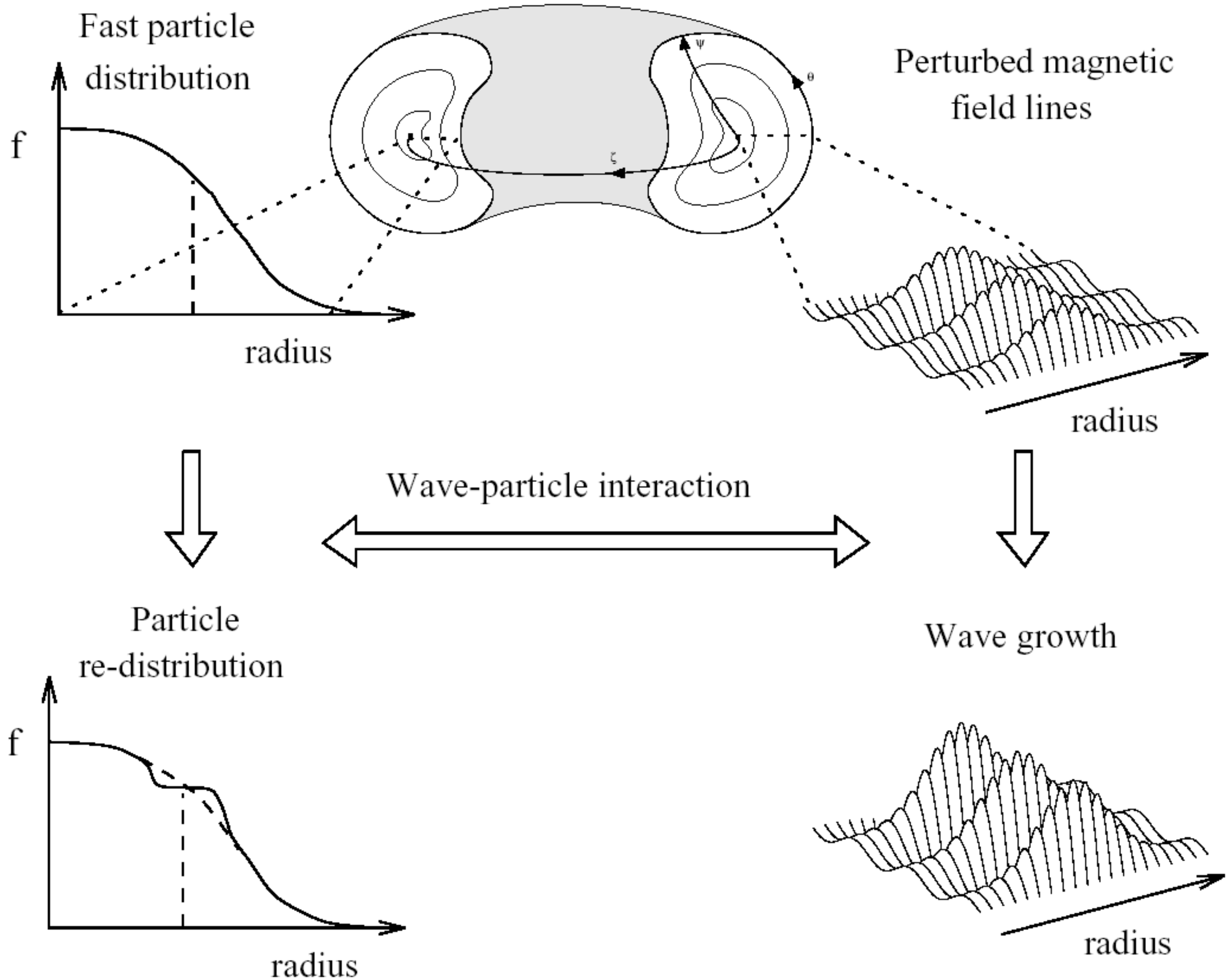
- Collective instabilities
  - Fast particle gradients act as source of free energy
    - Non-Maxwellian distribution
  - $\gamma \sim \omega\, \partial f/\partial E + n\, \partial f/\partial P_\phi$
    $\sim \omega\, \partial f/\partial E - n\, \partial f/\partial \psi$
  - Negative radial gradient
    $\Rightarrow$ *Drive (n>0)*
  - Negative energy gradient
    $\Rightarrow$ *Damping*





HOW CAN WE MODEL NONLINEAR FAST ION DRIVEN INSTABILITIES IN FUSION PLASMAS?

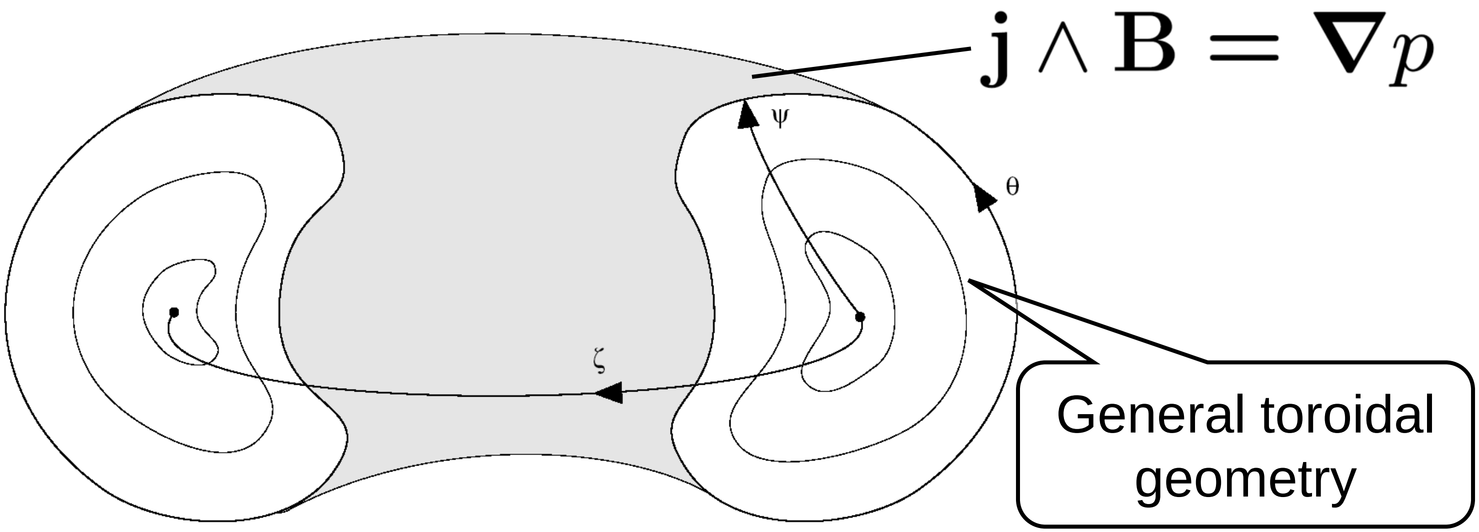
The HAGIS Code



S. D. Pinches *et al.*, Comput. Phys. Commun. **111** (1998)

Equilibrium Representation

- Straight field line (Boozer) coordinates ψ_p, θ, ζ



$$
\begin{aligned}
\mathrm{B} &= \delta(\psi_p, \theta)\nabla\psi_p + I(\psi_p)\nabla\theta + g(\psi_p)\nabla\zeta, \\
\mathrm{B} &= \nabla\psi \wedge \nabla\theta - \nabla\psi_p \wedge \nabla\zeta, \\
\Rightarrow \quad \mathrm{A} &= \psi\nabla\theta - \psi_p\nabla\zeta.
\end{aligned}
$$

Evolution of Energetic Particles

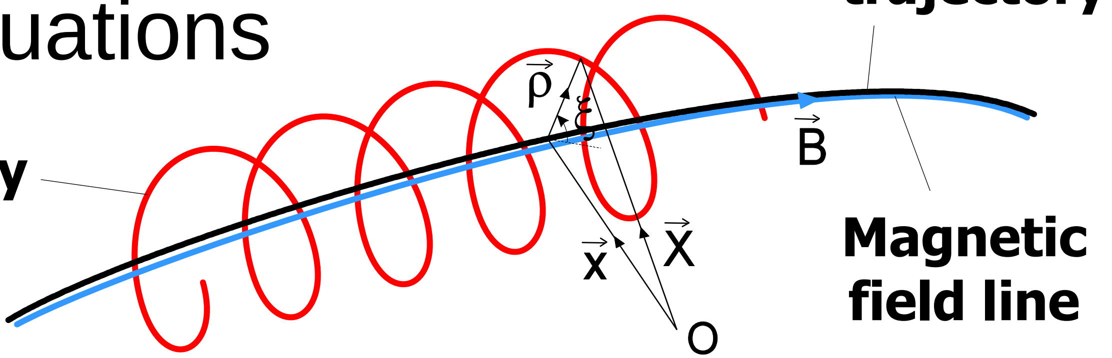
Exact particle Lagrangian, $\mathcal{L}_{exact} = \sum_{ep} \frac{1}{2} m V^2 + e\mathbf{V} \cdot \mathbf{A} - e\phi$

is gyro-averaged and written in the form,

$$\mathcal{L}_{ep} = \sum_{j=1}^{n_p} P_{\theta j}\dot{\theta}_j + P_{\zeta j}\dot{\zeta}_j - \mathcal{H}_j$$

with $$\mathcal{H}_j = \frac{1}{2} m_j v_{\parallel j}^2 + \mu_j B_j + e_j \phi_j$$

leading to $4 \times n_p$ equations



Equations of Motion

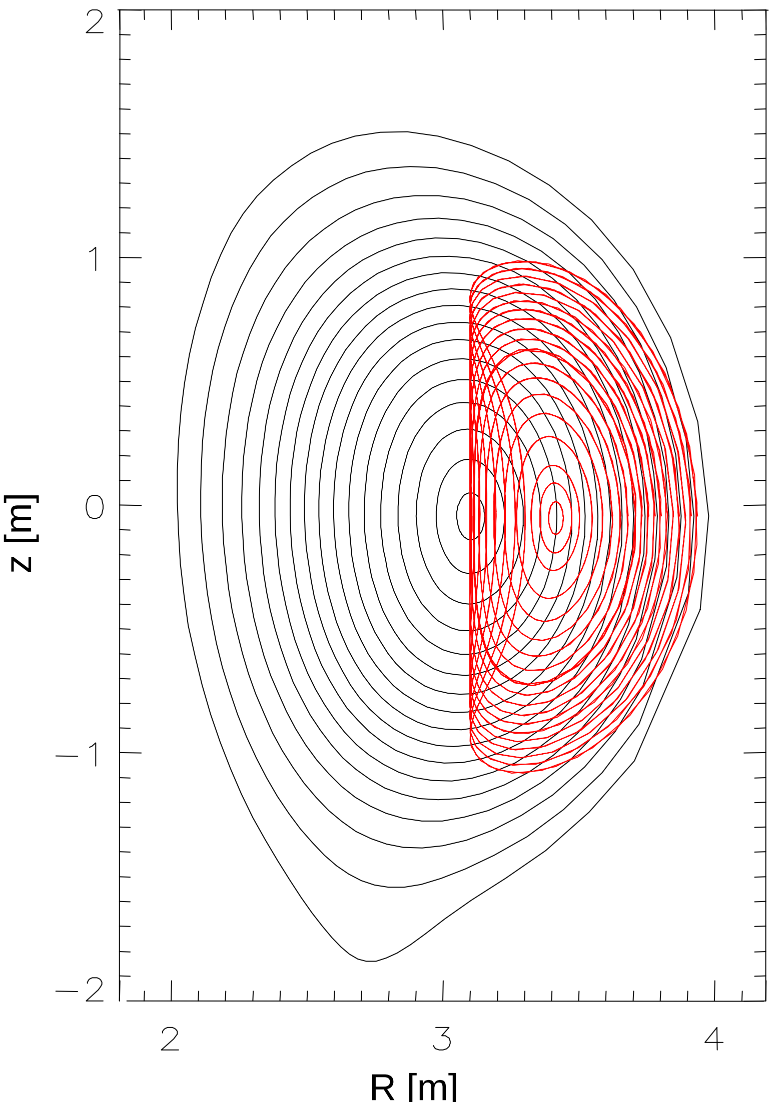
Derived from total system Hamiltonian for each particle:

$$
\begin{aligned}
\dot{\theta} &= \frac{1}{D}\left[\rho_{\|}B^2(1-\rho_c g' - g\tilde{\alpha}') + g\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\}\right], \\
\dot{\zeta} &= \frac{1}{D}\left[\rho_{\|}B^2(\rho_c I' + q + I\tilde{\alpha}') - I\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\}\right], \\
\dot{\psi}_p &= \frac{1}{D}\left[\rho_{\|}B^2\left(g\frac{\partial\tilde{\alpha}}{\partial\theta} - I\frac{\partial\tilde{\alpha}}{\partial\zeta}\right) - \left(g\frac{\partial\tilde{\Phi}}{\partial\theta} - I\frac{\partial\tilde{\Phi}}{\partial\zeta}\right) - g(\rho_{\|}^2 B + \mu)\frac{\partial B}{\partial\theta}\right], \\
\dot{\rho}_{\|} &= \frac{1}{D}\left[\left(I\frac{\partial\tilde{\alpha}}{\partial\zeta} - g\frac{\partial\tilde{\alpha}}{\partial\theta}\right)\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\} - (q + \rho_c I' + I\tilde{\alpha}')\frac{\partial\tilde{\Phi}}{\partial\zeta}\right. \\
&\quad \left. + (\rho_c g' - 1 + g\tilde{\alpha}')\left\{(\rho_{\|}^2 B + \mu)\frac{\partial B}{\partial\theta} + \frac{\partial\tilde{\Phi}}{\partial\theta}\right\}\right] - \frac{\partial\tilde{\alpha}}{\partial t},
\end{aligned}
$$

RB White & MS Chance, Phys. Fluids 27 10 (1984)

Fast Particle Orbits

- ICRH ions in JET deep shear reversal
  - On axis heating†: $\Lambda = \mu B_0/E = 1$
  - E = 500 keV
- Produces predominately potato orbits
- Particle trajectories verified through comparison with other codes and analytic solutions



†J. Hedin, PhD Thesis 1999

Calculation of AE Eigenfunctions

Wave Lagrangian:

$$\mathcal{L}_w = \sum \left[\frac{1}{2}mv^2 + e\left(\mathbf{A}\cdot\mathbf{v} - \phi\right)\right] + \frac{1}{2\mu_0}\int_V \left(\frac{1}{c^2}E^2 - B^2\right) dx^3$$

Expanding in perturbed field powers:

- $\mathcal{L}^{(0)}$ describes the equilibrium and is solved by, for example, HELENA
- $\mathcal{L}^{(1)}$ describes first order force balance
- $\mathcal{L}^{(2)}$ describes fixed amplitude Alfvén Eigenmodes and is solved by appropriate linear codes, e.g. CASTOR, MISHKA, PHOENIX, or LIGKA

Wave Evolution

- Linear eigenmode structure is assumed to remain fixed throughout simulations
- Each wave is allowed two degrees of freedom, amplitude and phase-shift; $\mathcal{A}_k$ and α_k

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

- The wave Lagrangian can then be written as

$$L_w = \sum_{k=1}^{n_w} \frac{E_k}{\omega_k} A_k^2 \dot{\alpha}_k,$$

where

$$E_k = \frac{1}{2\mu_0} \int_V \frac{\left|\nabla_\perp \tilde{\Phi}_k\right|^2}{v_A^2} d^3x,$$

and n_w is the number of eigenmodes in the system

Wave Equations

- Linear eigenstructure assumed invariant
- Introduce slowly varying amplitude and phase:

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

- Gives wave equations as:

Additional mode damping rate, γ_d

$$\dot{\mathcal{X}}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta\Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) S_{jkm} + \mathcal{X}_k \gamma_d,$$

$$\dot{\mathcal{Y}}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta\Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) C_{jkm} + \mathcal{Y}_k \gamma_d,$$

- where

$$\mathcal{X}_k \equiv A_k \cos(\alpha_k), \qquad \mathcal{Y}_k \equiv A_k \sin(\alpha_k),$$

$$C_{jkm} \equiv \Re e[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$

$$S_{jkm} \equiv \Im m[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$

$$\Theta_{jkm} \equiv n_k \zeta_j - m\theta_j - \omega_k t$$

Distribution Function

- Represented by a finite number of *markers*
- Markers represent deviation from initial distribution function - so-called δf method
  - Dramatically reduces numerical noise

$$f = \underbrace{f_0(\mathcal{E}, P_\zeta; \mu)}_{\text{analytic}} + \underbrace{\delta f(\Gamma^{(p)}, t)}_{\text{markers}}$$

$$\frac{df}{dt} = 0 \Rightarrow \dot{\delta f} = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}} - \nu_{eff}\,\delta f$$

$$\int f\, g\, d\Gamma^{(p)} \longleftrightarrow \int f_0\, g\, d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j\, g_j$$

where $\delta n_j(t) \equiv \delta f_j(t)\, \Delta\Gamma_j^{(p)}(t)$

SE Parker and WW Lee, Phys. Fluids B 5 (77) (1993)
RD Denton and M Kotschenreuther, J. Comp. Phys. 119 (1995)

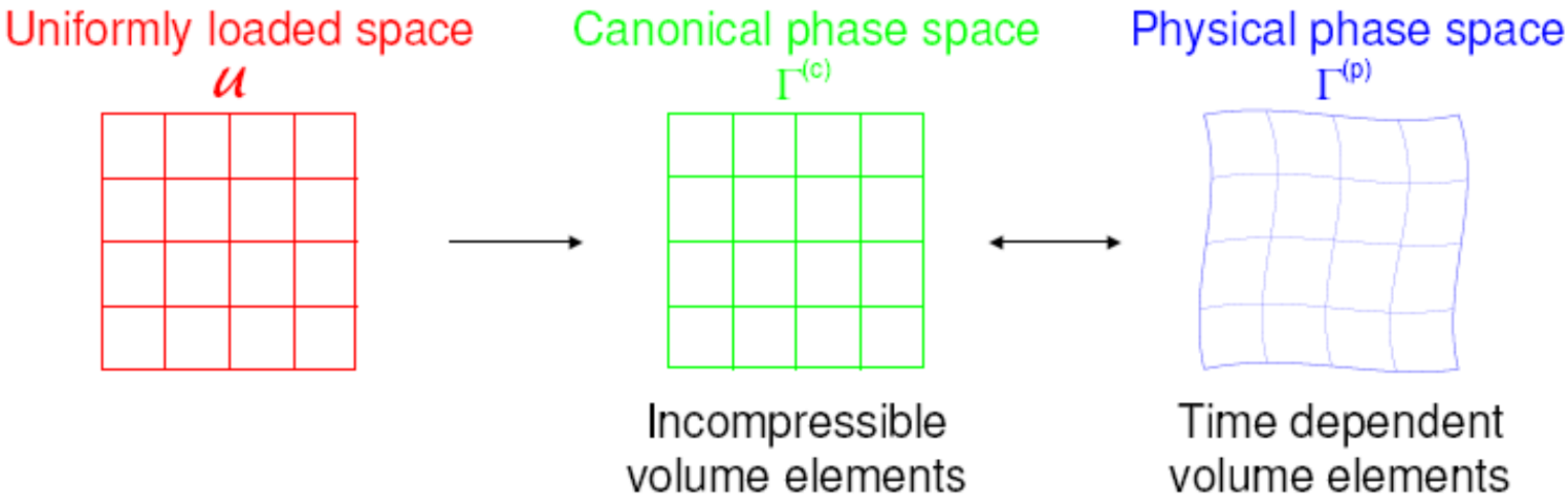
Marker Loading

- Number of particles represented by a marker:

$$\delta n_j(t) \equiv \delta f_j(t)\, \Delta\Gamma_j^{(p)}(t)$$

- Physical volume element associated with a marker:

$$\Delta\Gamma_j^{(p)} \equiv \mathcal{J}_j^{(pc)}(t)\, \underbrace{\mathcal{J}_j^{(cu)}(0)\, \Delta\mathcal{U}_j}_{\Delta\Gamma_j^{(c)}(0)},$$



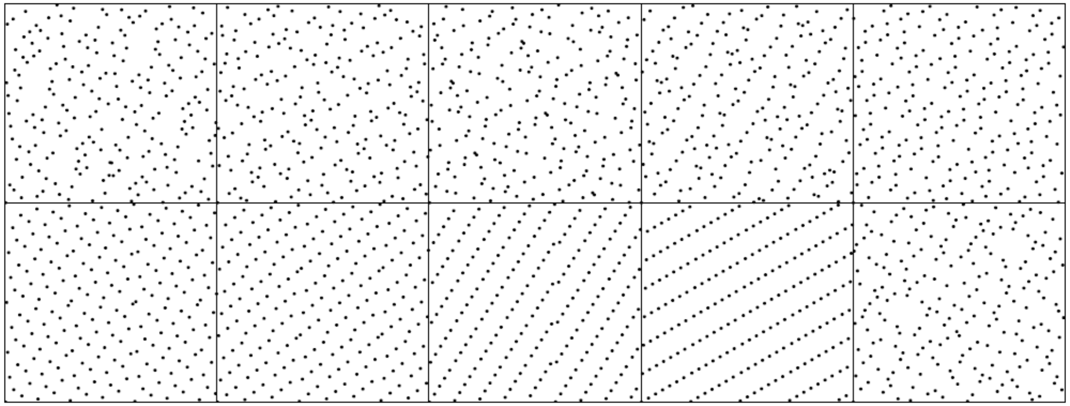
Quiet Start Method

- Markers are uniformly loaded using Hammersley's sequence:

$$x_i = \{i/N, \phi_2(i), \phi_3(i), \phi_5(i), \phi_7(i)\}.$$

- If integer i is written in base r:

$$\begin{aligned} i &= a_0 + a_1 r + a_2 r^2 + \cdots \\ \phi_r(i) &= a_0 r^{-1} + a_1 r^{-2} + a_2 r^{-3} + \cdots \end{aligned}$$



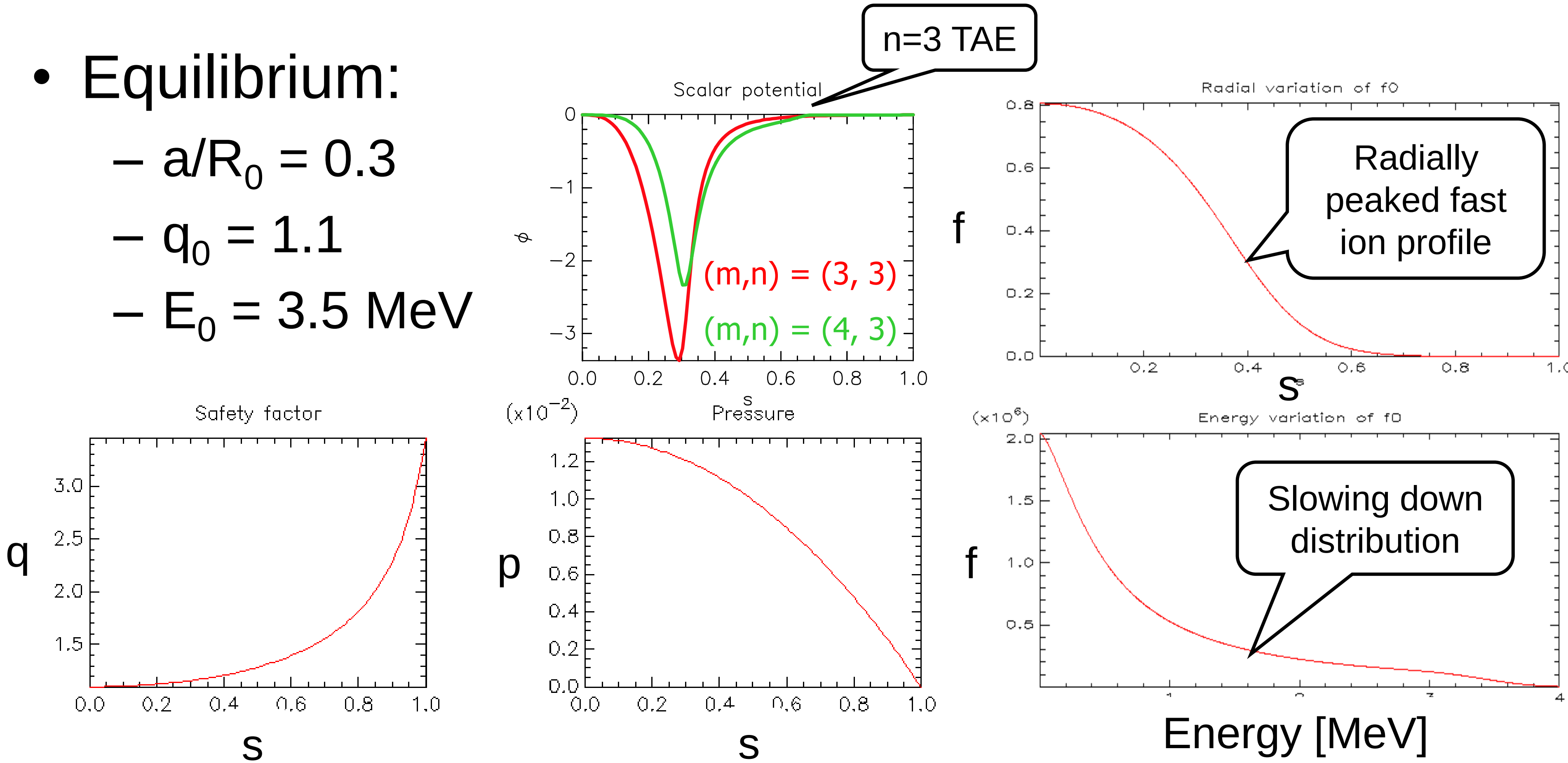
Projections of uniformly loaded 5-D hypercube

- This achieves a discrepancy $\propto\ 1/N$, where a random distribution has a discrepancy $\propto 1/\sqrt{N}$.

J. H. Halton, Numer. Math. 2 (1960), 84{90 and 196

Example of Linear Growth and Saturation of a TAE

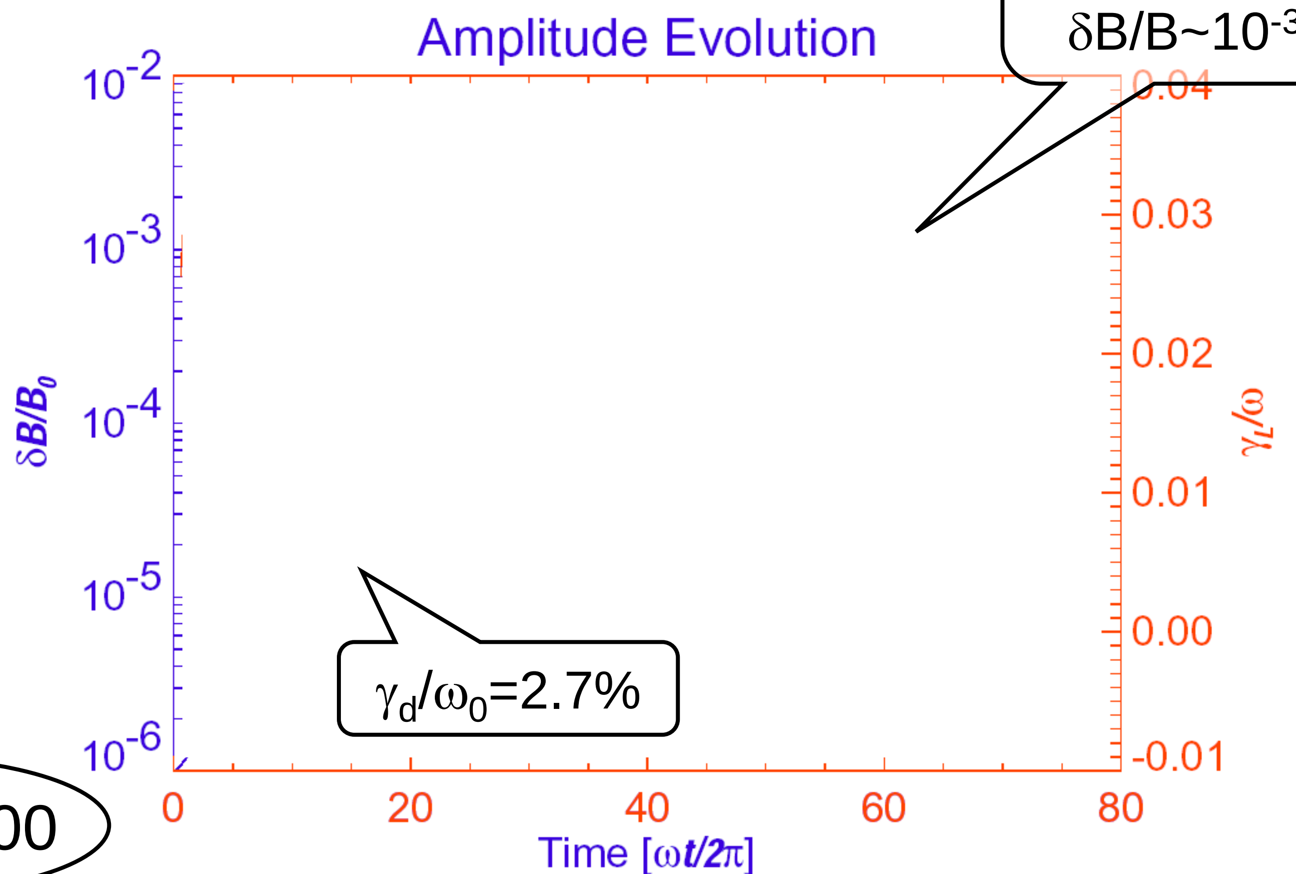
- Equilibrium:
  - $a/R_0 = 0.3$
  - $q_0 = 1.1$
  - $E_0 = 3.5$ MeV



S. D. Pinches *et al.*, Comput. Phys. Commun. **111** (1998)

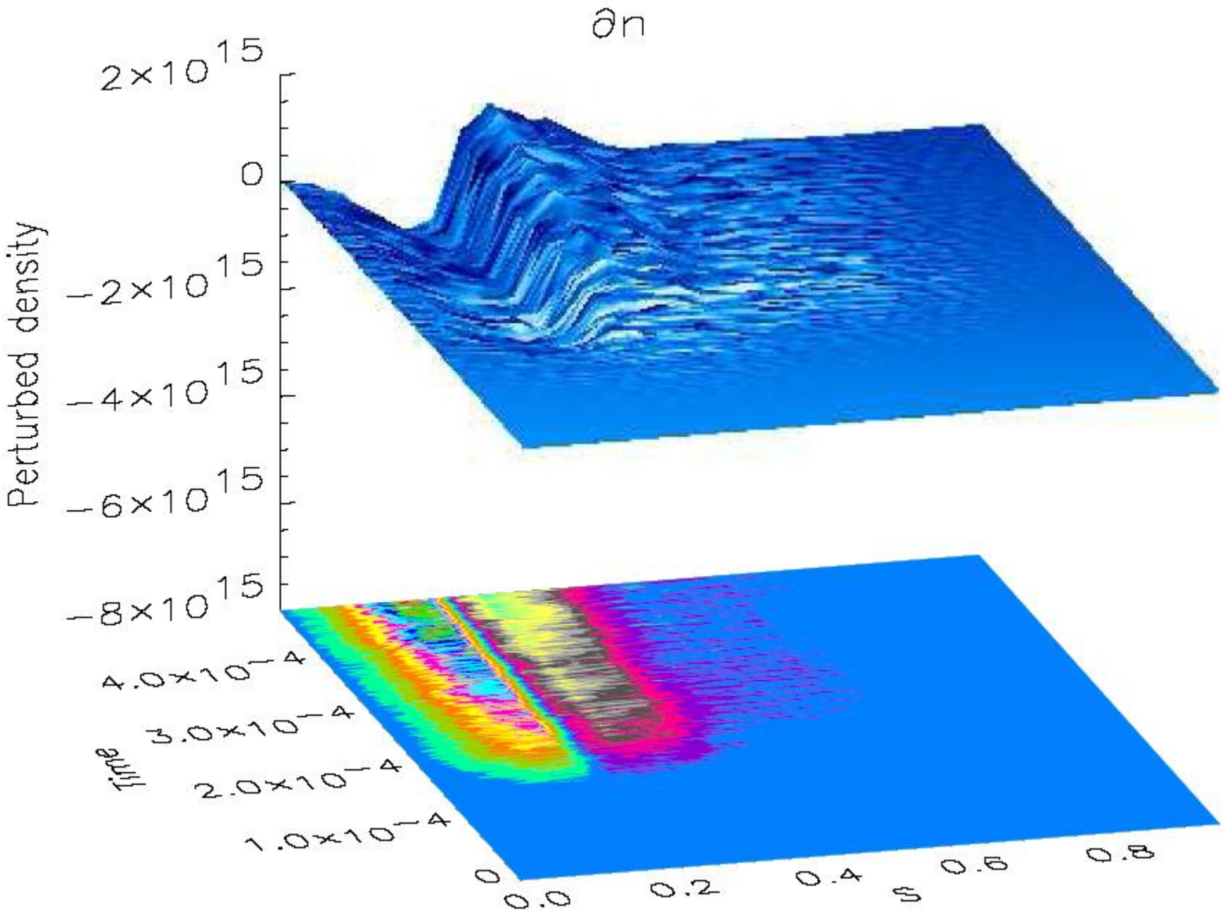
Linear Growthrate

- $\langle \beta_f \rangle = 3\times10^{-4}$



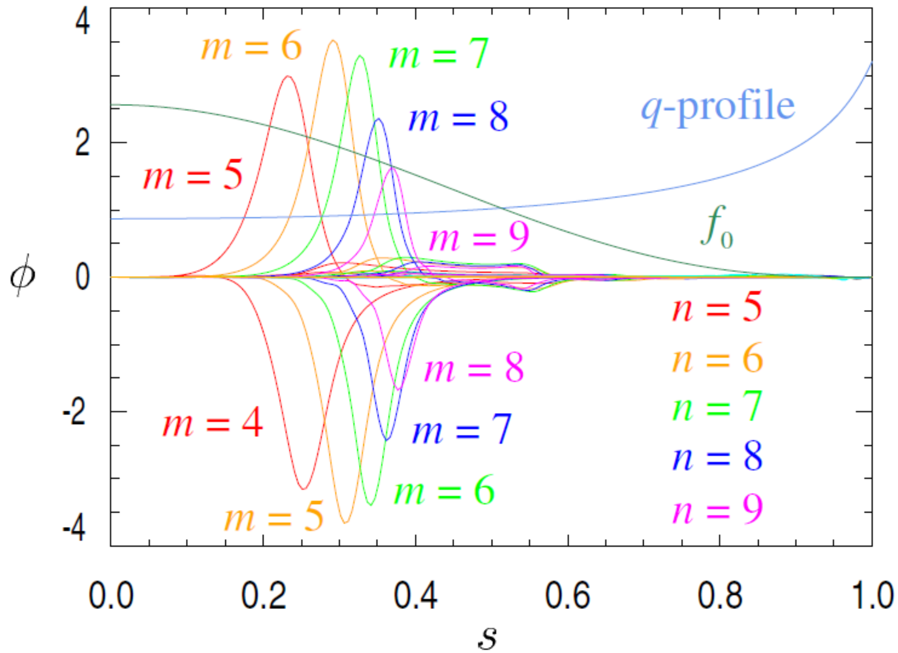
n_p = 52,500

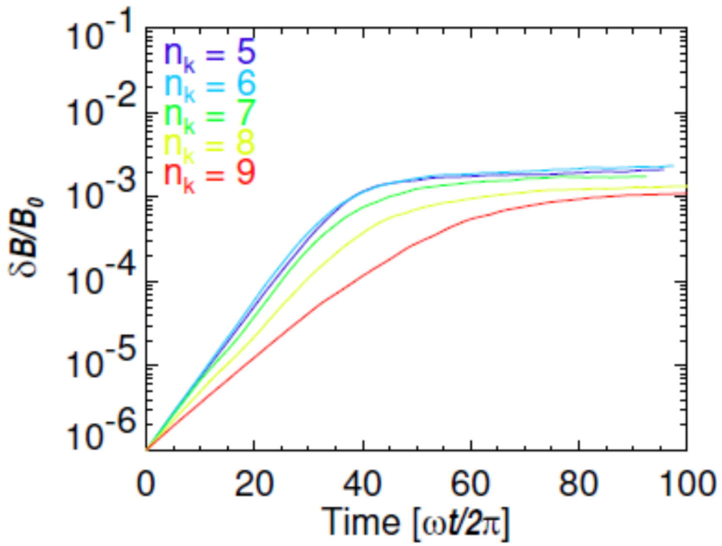
Fast Ion Redistribution due to TAE

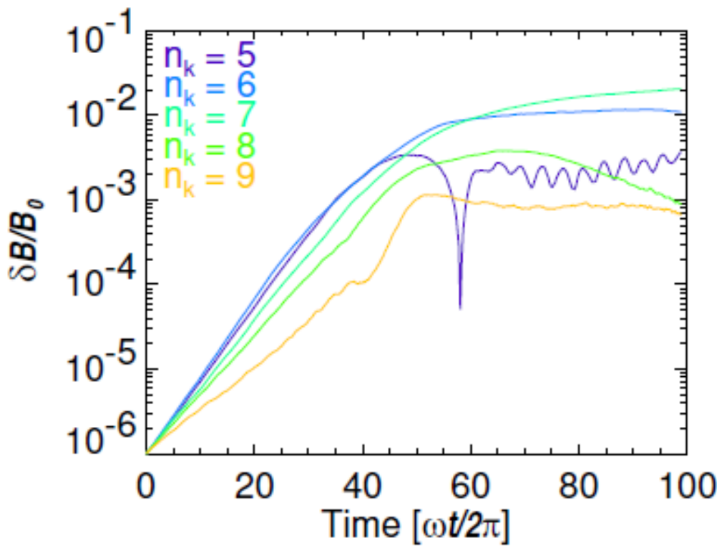


Multiple KTAE in JET

- Multiple KTAE ($n = 5 – 9$) in JET interacting through the driving alpha particle distribution







INCLUDING DISSIPATION

Nonlinear Theory and Dissipative Effects

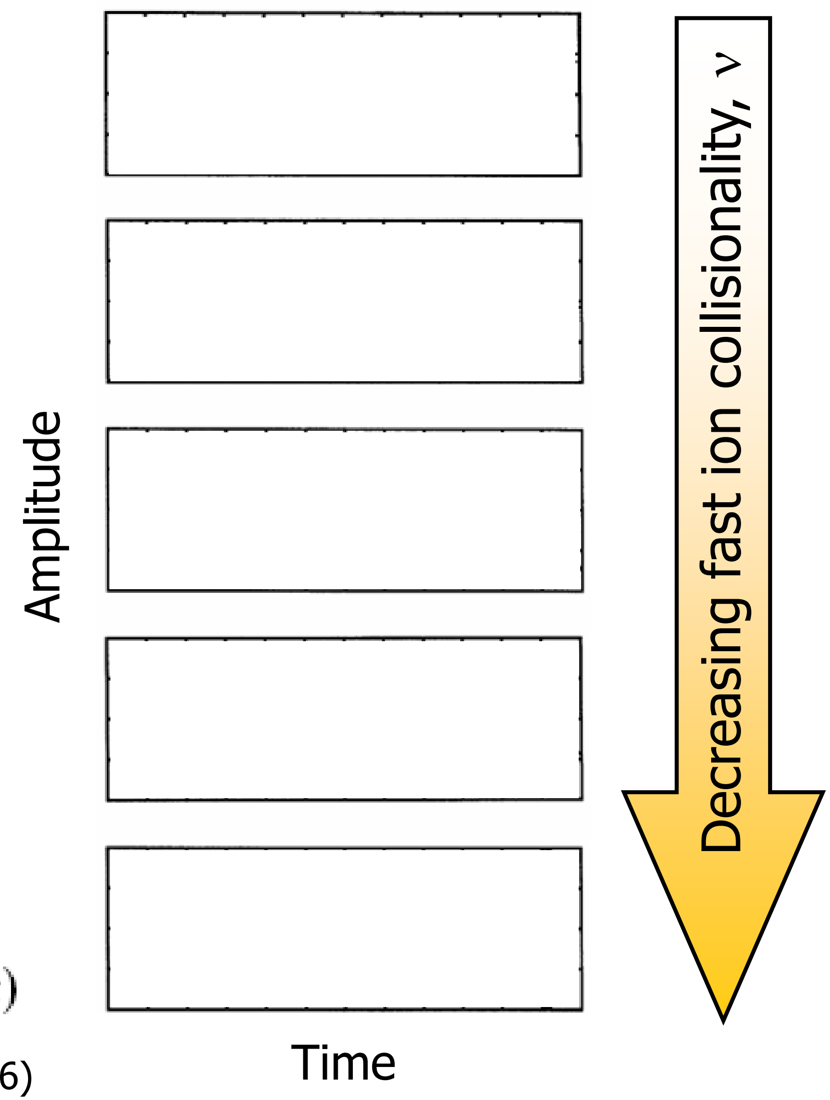
- When modes are near marginal stability then there are various competing effects
  - Drive from fast ions, γ_L
  - Damping from background plasma, γ_D
  - Reconstitution of profiles, ν_{eff}

$$|\gamma_L - \gamma_D| \sim \nu_{eff} << \gamma_L, \gamma_D$$

Nonlinear Theory

- Nonlinear cubic equation describes Alfvén eigenmodes near threshold
  - ν is the collision frequency for fast particles

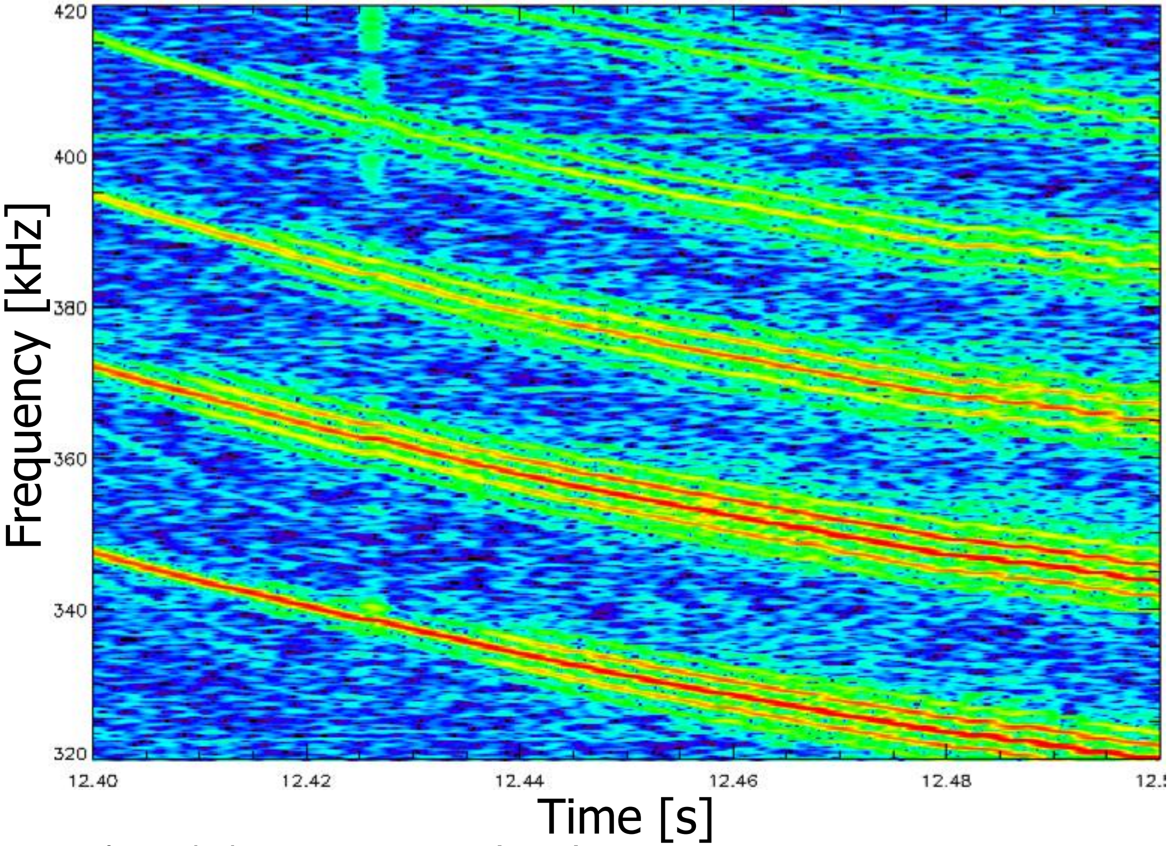
$$\frac{dA}{d\tau} = A(\tau) - \frac{1}{2}\int_0^{\tau/2} dz\, z^2 A(\tau - z) \times \int_0^{\tau-2z} dx\, \exp[-\hat{\nu}(2z + x)] \times A(\tau - z - x)A(\tau - 2z - x)$$

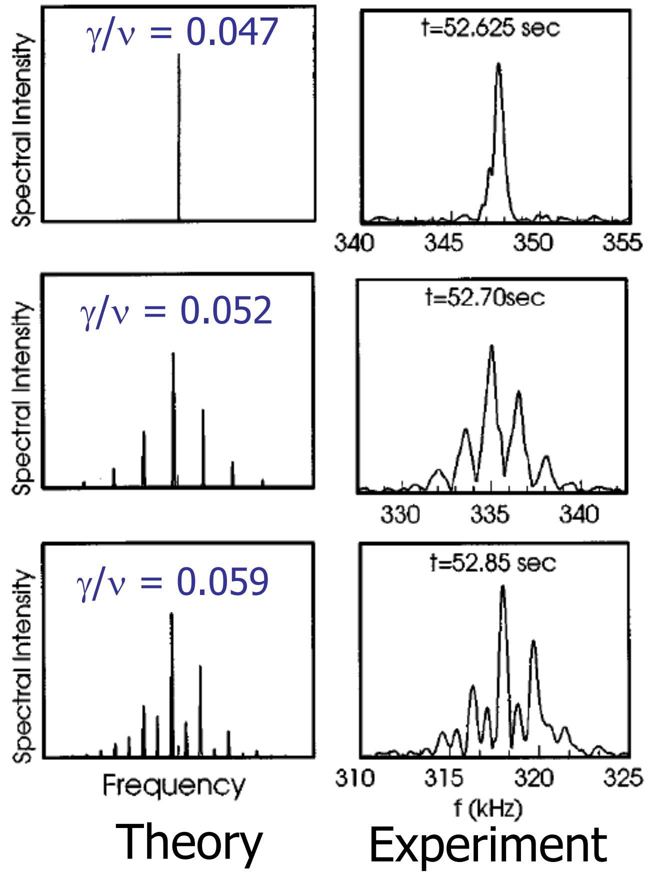


H.L. Berk, B. N. Breizman & M. Pekker. *Phys. Rev. Lett.* **76** (1996)

Closer look at TAE…

- Resonant particles relax through collisions
- Single mode undergoes pitchfork splitting
  - Used to determine γ and ν

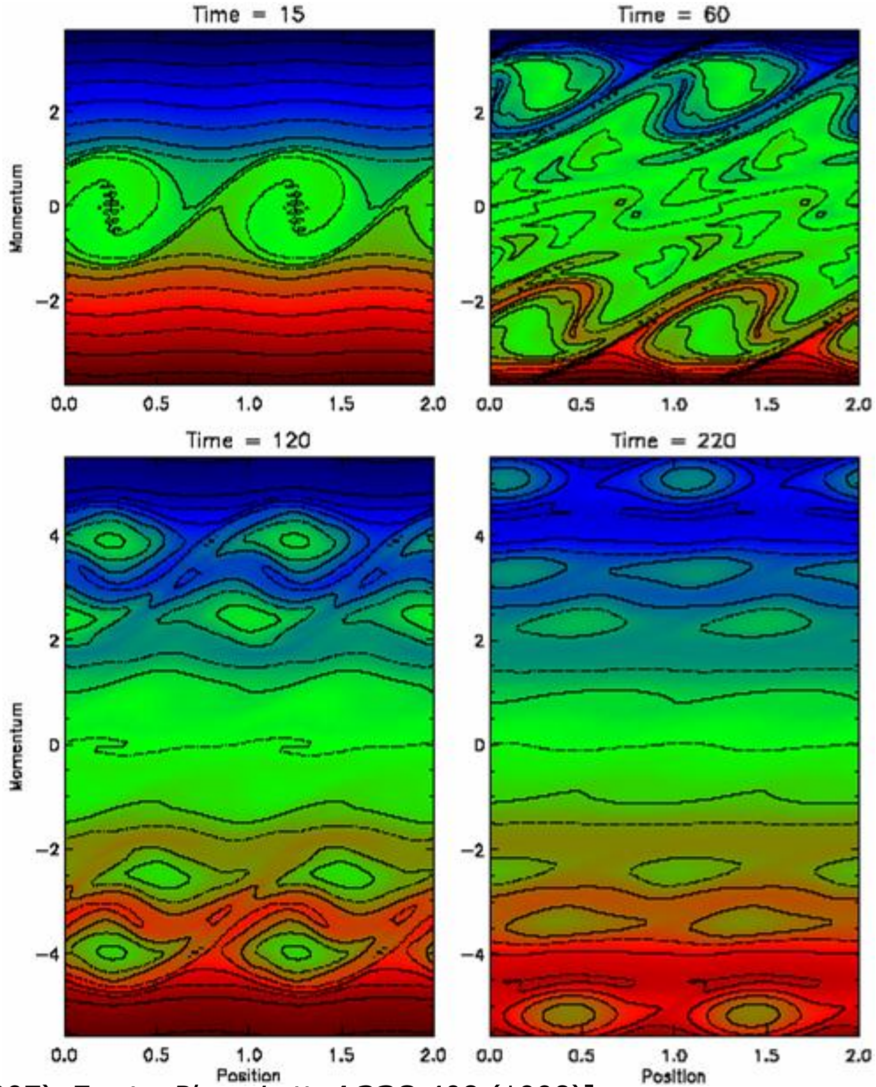




A. Fasoli *et al*. Phys. Rev. Lett. **81** (1998)

Frequency Sweeping

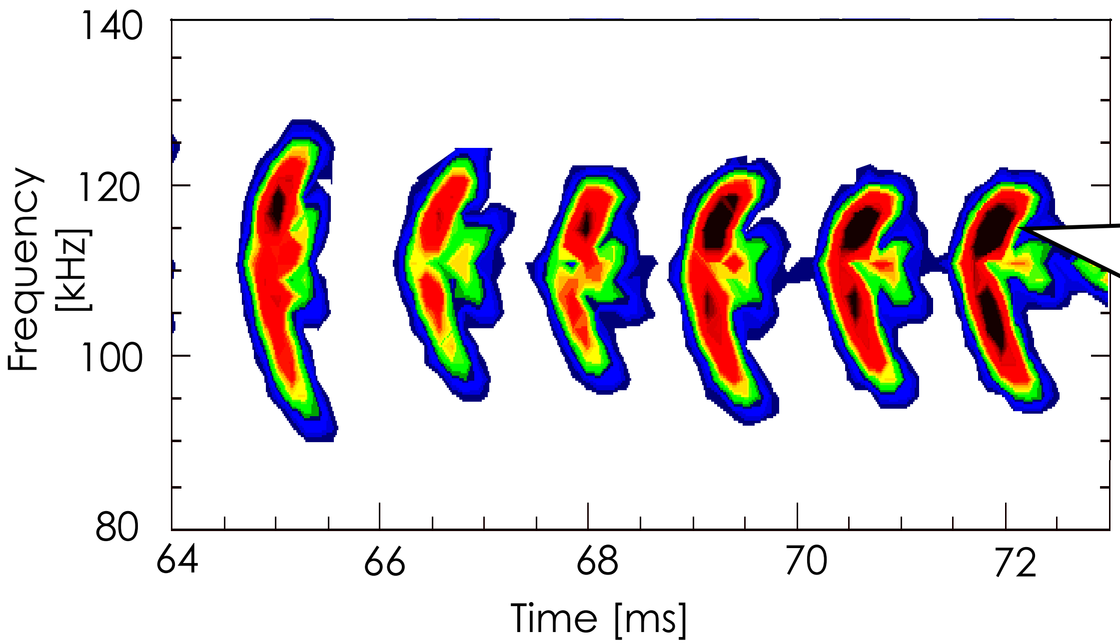
- Occurs when mode is close to marginality
  - Damping balancing drive
- Structures form in fast particle distribution function
  - Holes and clumps
- These support long-lived nonlinear BGK waves
- Background dissipation is balanced by frequency sweeping



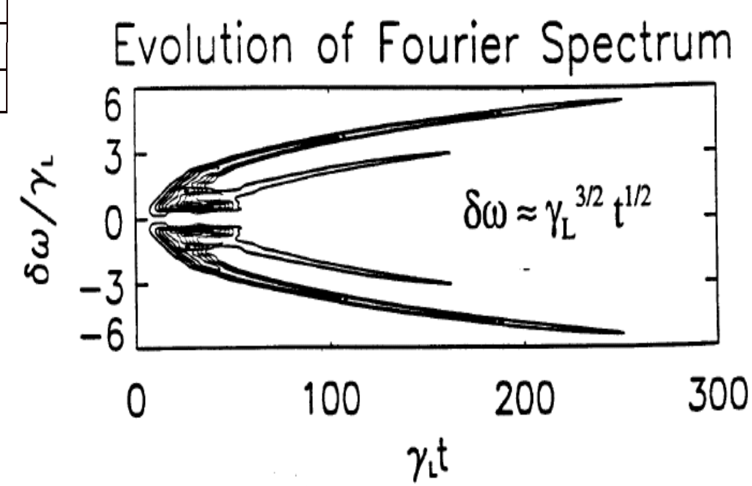
[H.L. Berk, B.N. Breizman & N.V. Petviashvili, *Phys. Lett. A* **234** 213 (1997), Errata *Phys. Lett. A* **238** 408 (1998)]

Experimental Observations

- Frequency sweeping in MAST #5568

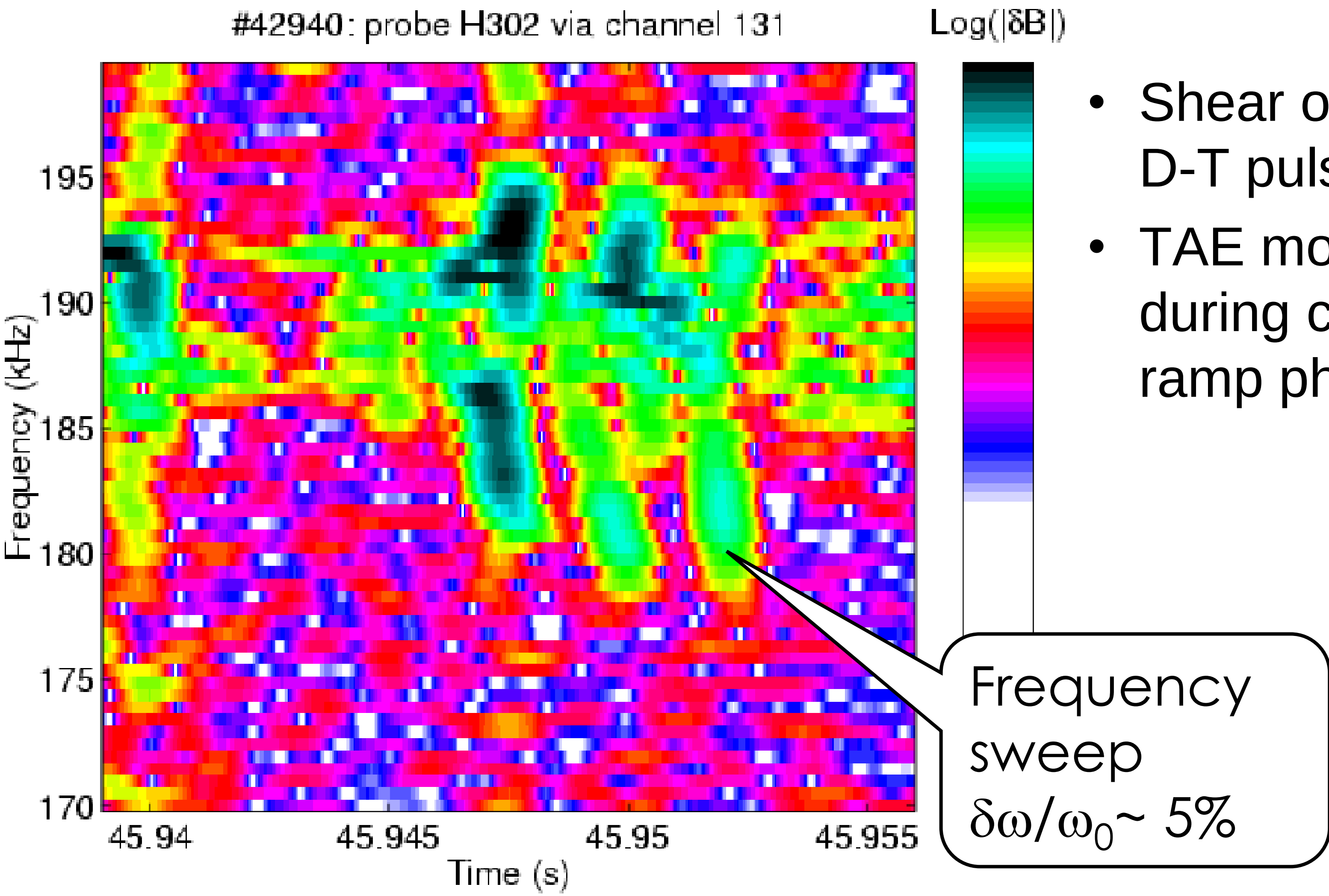


Simultaneous upwards and downwards frequency sweeping, $\delta\omega/\omega_0 \sim 20\%$



H.L. Berk, B.N. Breizman & N.V. Petviashvili. *Phys. Lett. A* **234** (1997), *Phys. Lett. A* **238** (1998)

JET Observations



- Shear optimised D-T pulse
- TAE modes during current ramp phase

Using Theory for Diagnostic Purposes

- Trapping frequency is related to TAE amplitude

$$\omega_{b,l}(t) \propto |\delta B|^{1/2}$$

- Frequency sweep is related to trapping frequency

$$\delta\omega \propto \omega_b^{3/2} t^{1/2}$$

- Amplitude related to frequency sweep

[Berk, Breizman & Petviashvili, Phys. Lett. A **234** 213 (1997)]

$$\frac{\delta B}{B} = \frac{1}{C_1^2}\left(\frac{\delta\omega^2}{C_2^2 t}\right)^{2/3}$$

Analytic estimates give correct order of magnitude. Numerical simulation required for more accurate estimate.

[H.L. Berk, B. N. Breizman & M. Pekker. *Phys. Rev. Lett.* **76** (1996)]

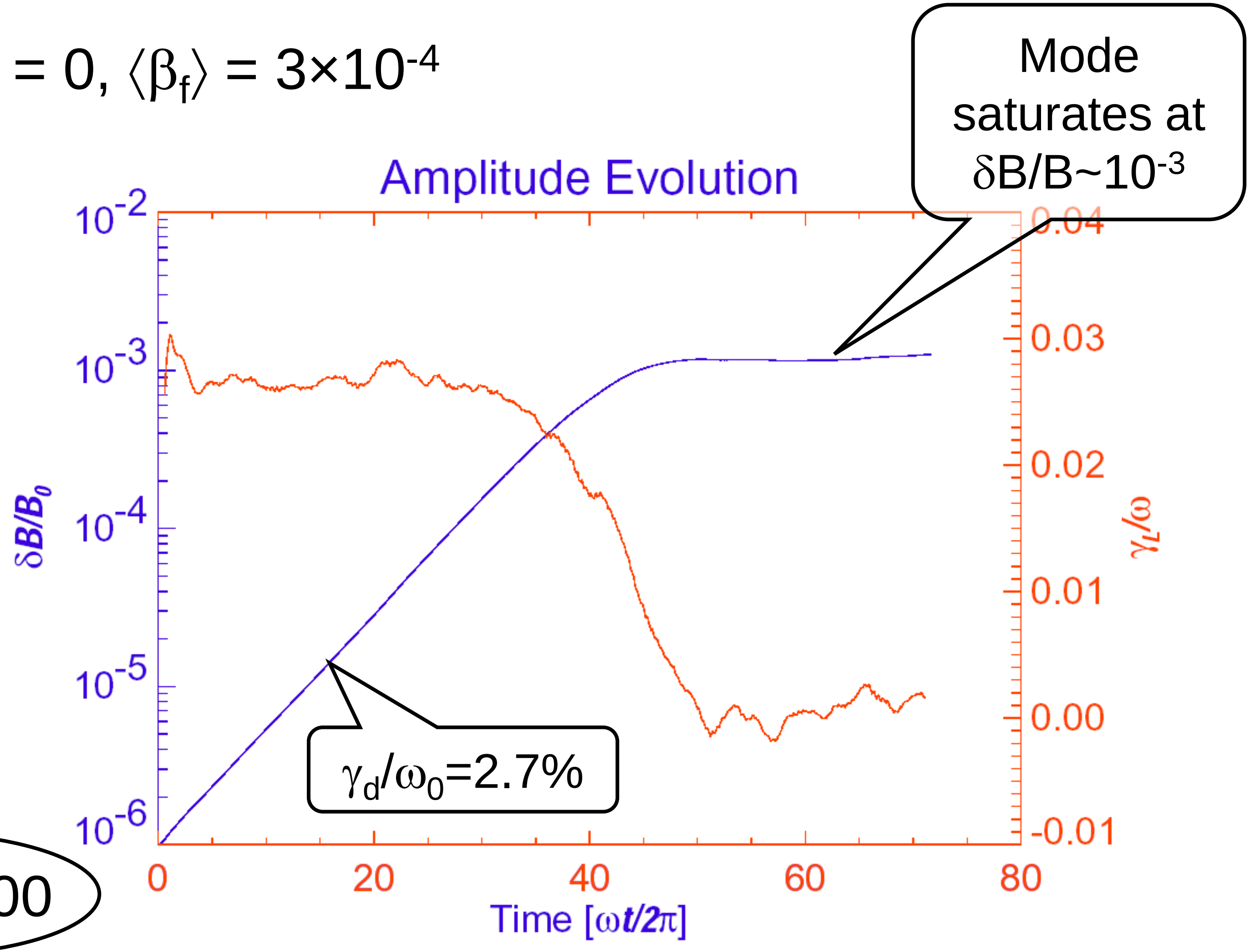
[S D Pinches *et al.*, *Plasma Phys. Control. Fusion* **46** S47-S57 (2004)]

Validation of Nonlinear Modelling

- Use experimentally observed rate of frequency sweeping to determine wave amplitude and compare with independent measurements
  - In general, numerical modelling is needed to establish the form factor that relates $\delta\omega$ and δB
  - Verify HAGIS for model case
  - Employ HAGIS to establish δB in general case
    - General geometry (including tight-aspect ratio)
    - Mode structure: global mode analysis

Recall *n* = 3 TAE example

- $\gamma_d/\omega_0 = 0$, $\langle\beta_f\rangle = 3\times10^{-4}$

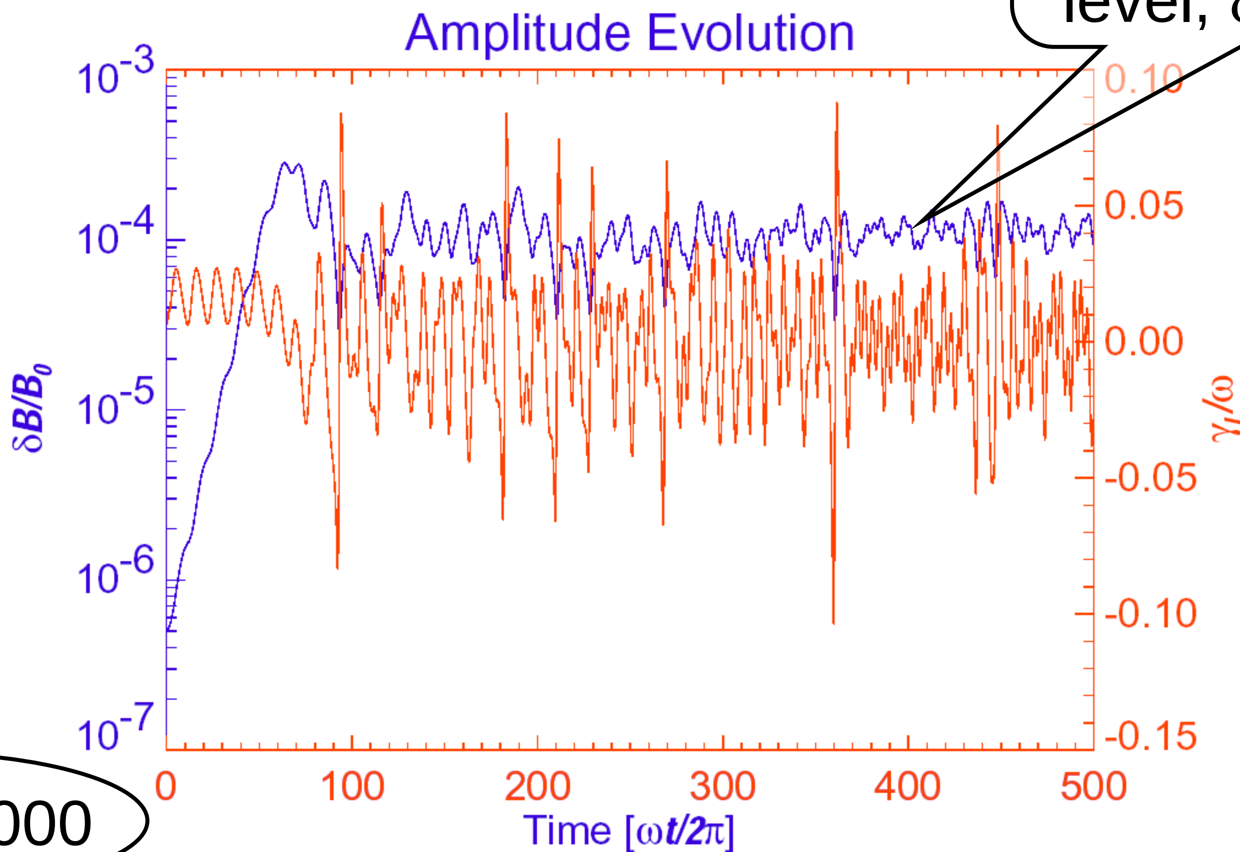


n_p = 52,500

…with additional damping

- γ_d/ω_0 = 2%, $\langle\beta_f\rangle = 3\times10^{-4}$

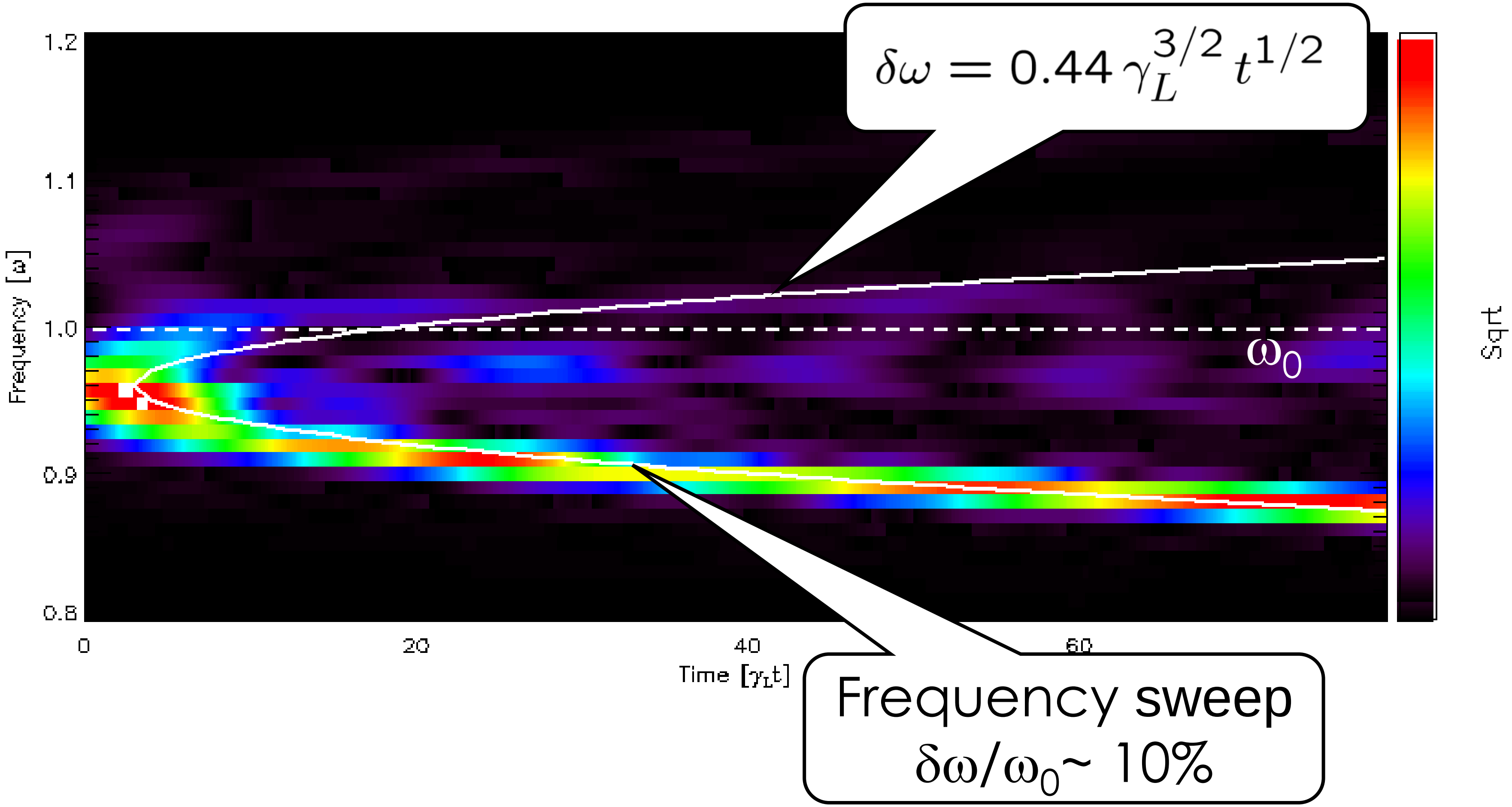
Mode saturates at much lower level, $\delta B/B \sim 10^{-4}$



n_p = 210,000

Frequency Sweeping

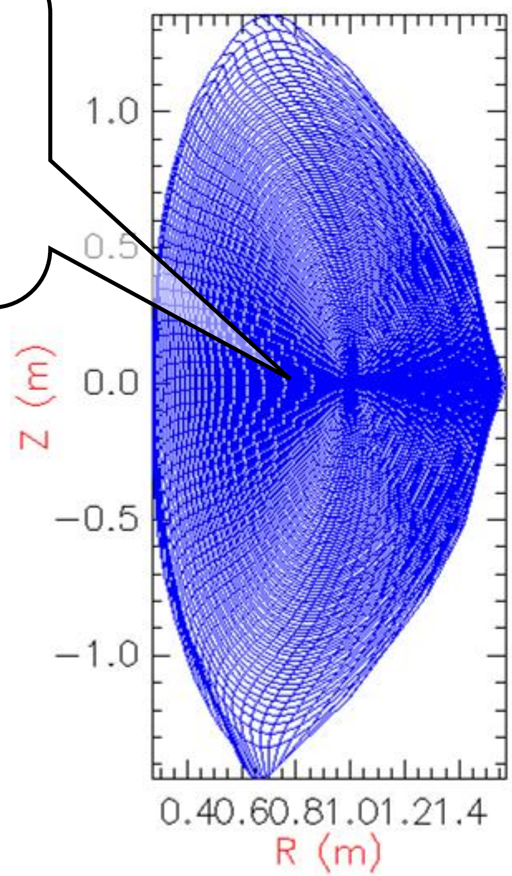
- Fourier spectrum of evolving mode

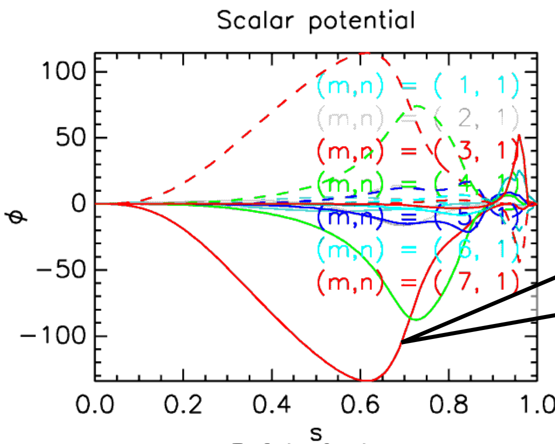


MAST #5568

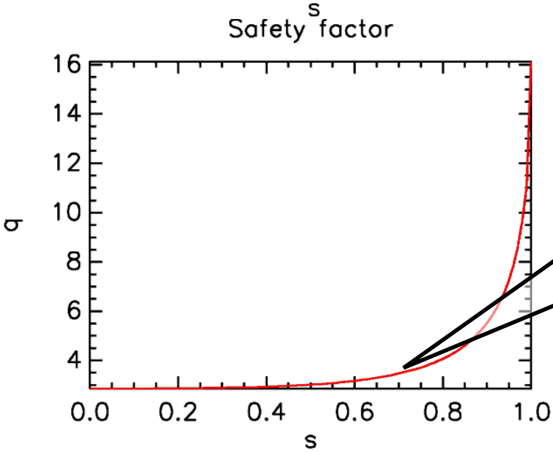
- Obtain factor relating ω_b and δB

E_b = 40 keV
a/R_0 = 0.7
B_0 = 0.5 T
R_0 = 0.77 m





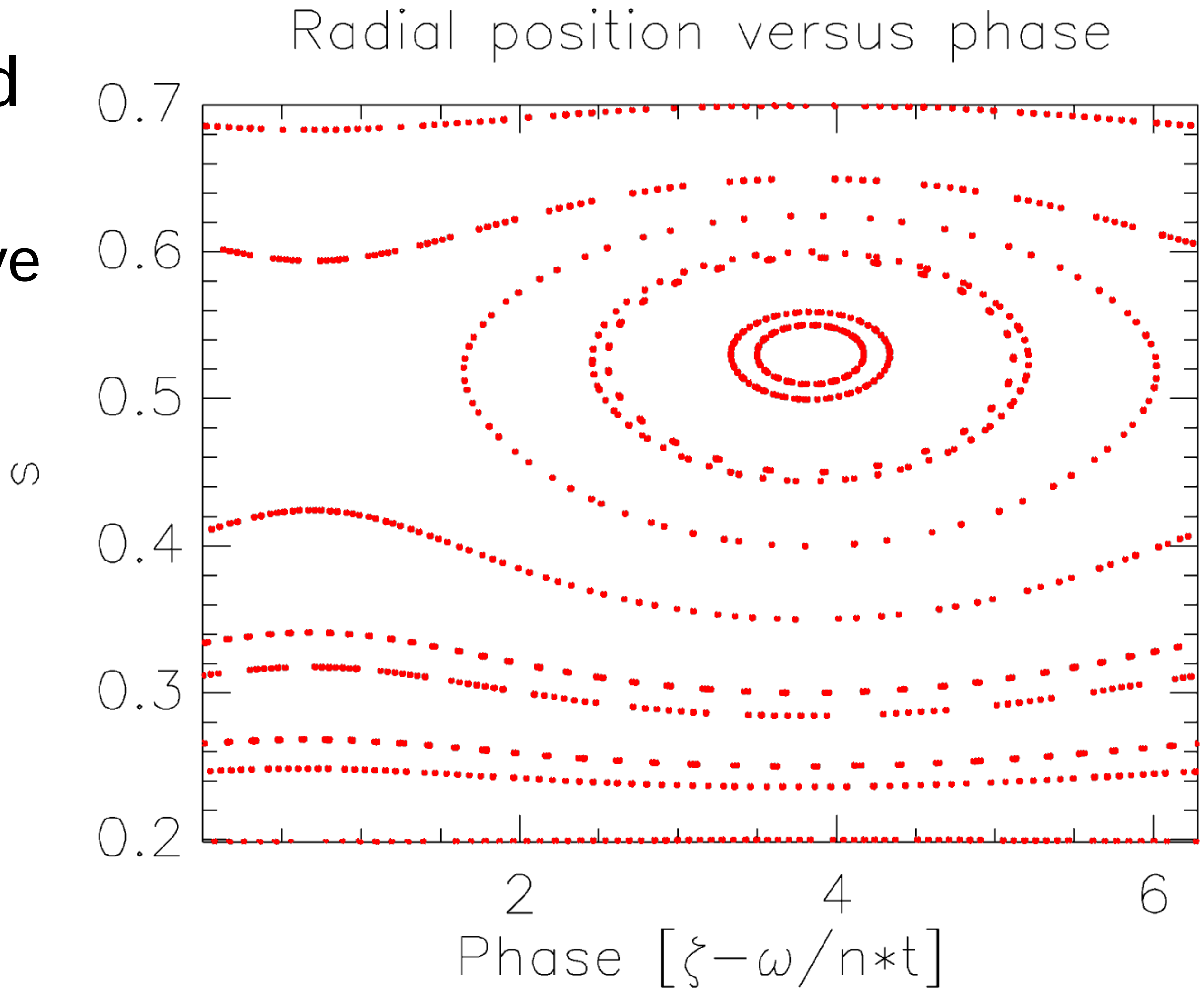
Global n=1 TAE



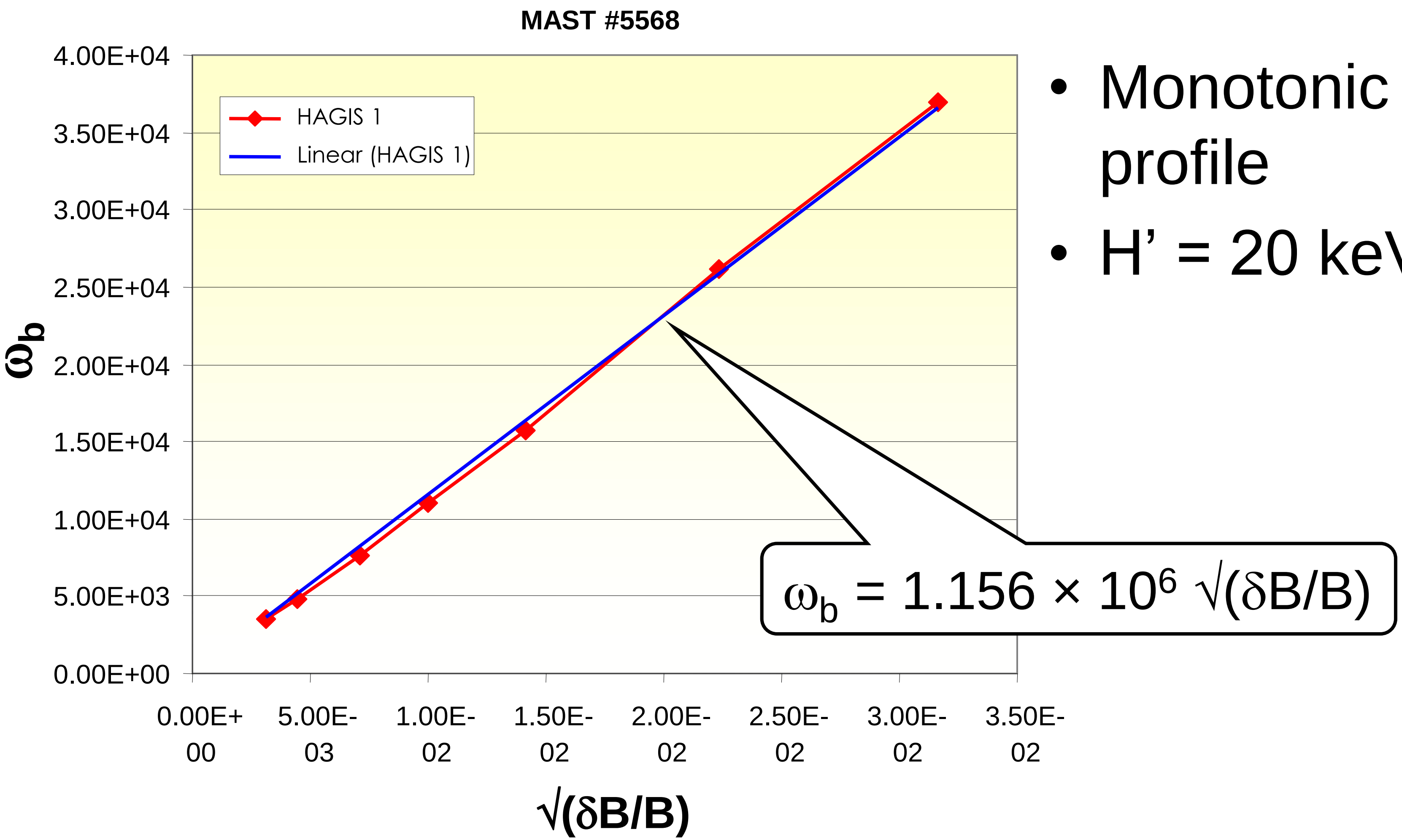
Monotonic q-profile

Particle Trapping in MAST

- Particles trapped in TAE wave
  - All particles have same
    $H' = E - \omega/n\, P_\zeta$
    $= 20$ keV
  - TAE amplitude:
    $\delta B/B = 10^{-3}$

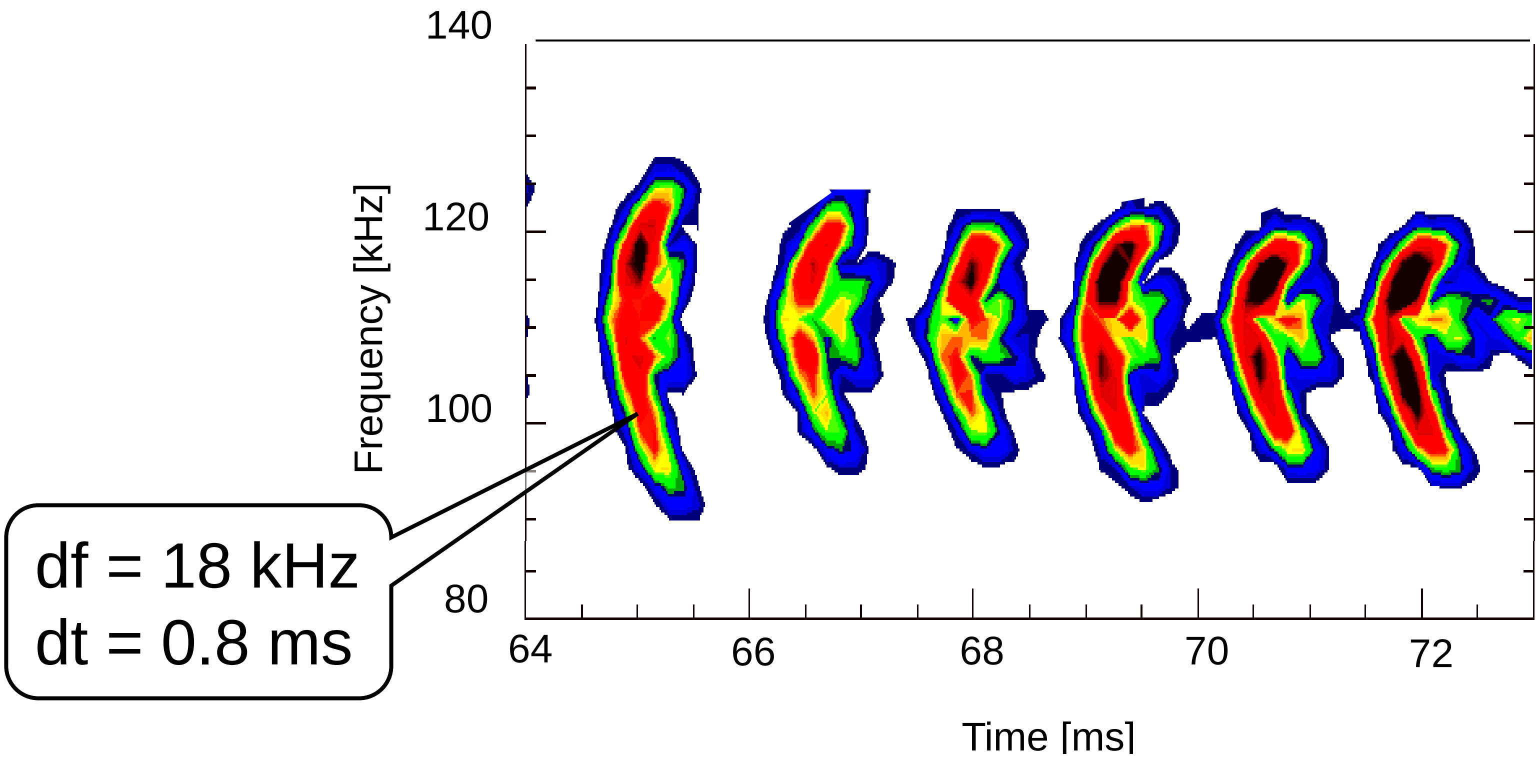


Scaling of Nonlinear Bounce Frequency



- Monotonic *q* profile
- H’ = 20 keV

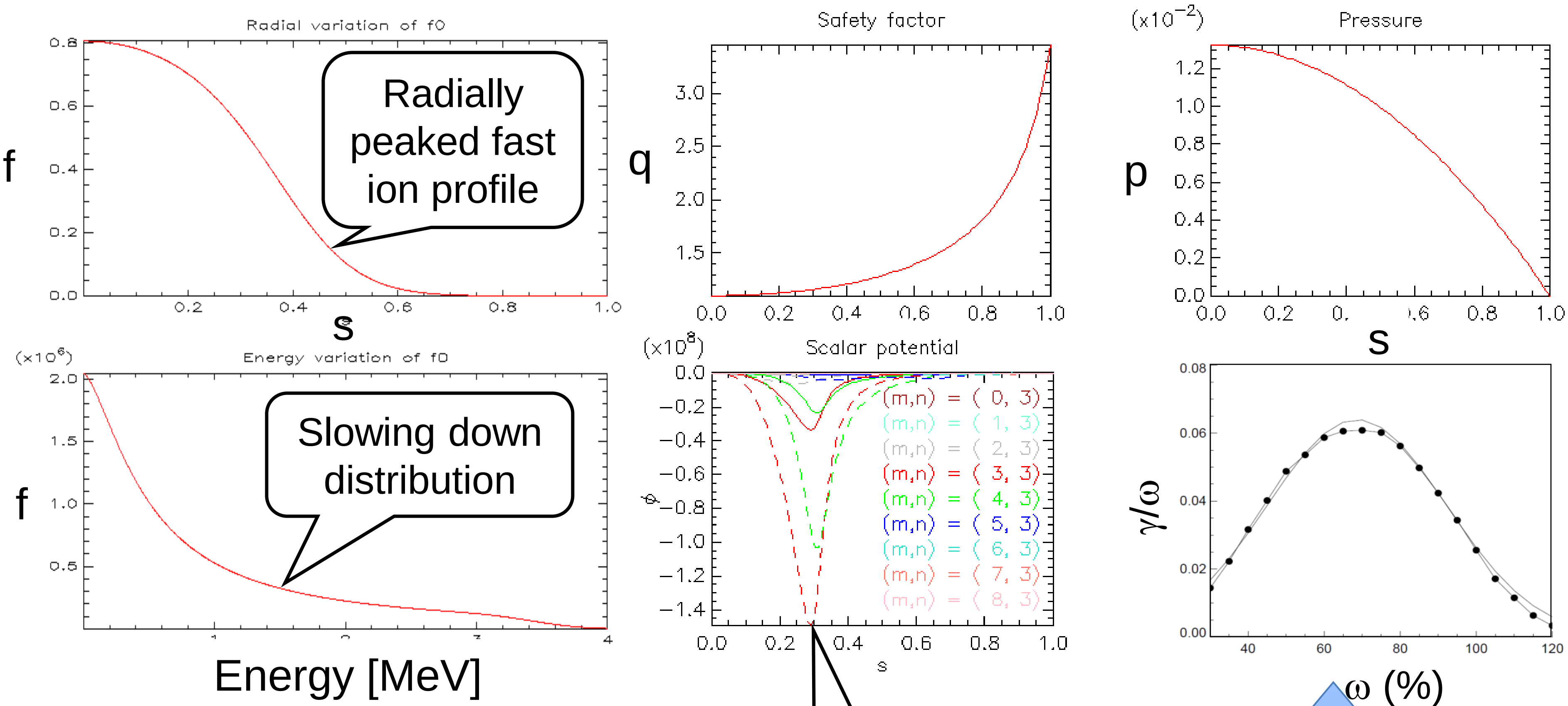
TAE Amplitude in MAST



$$\frac{\delta B}{B} = \frac{1}{(1.156 \times 10^6)^2}\left(\frac{32\,\delta f^2}{\delta t}\right)^{2/3}$$

$$= 4 \times 10^{-4}$$

Consider again our n = 3 TAE case



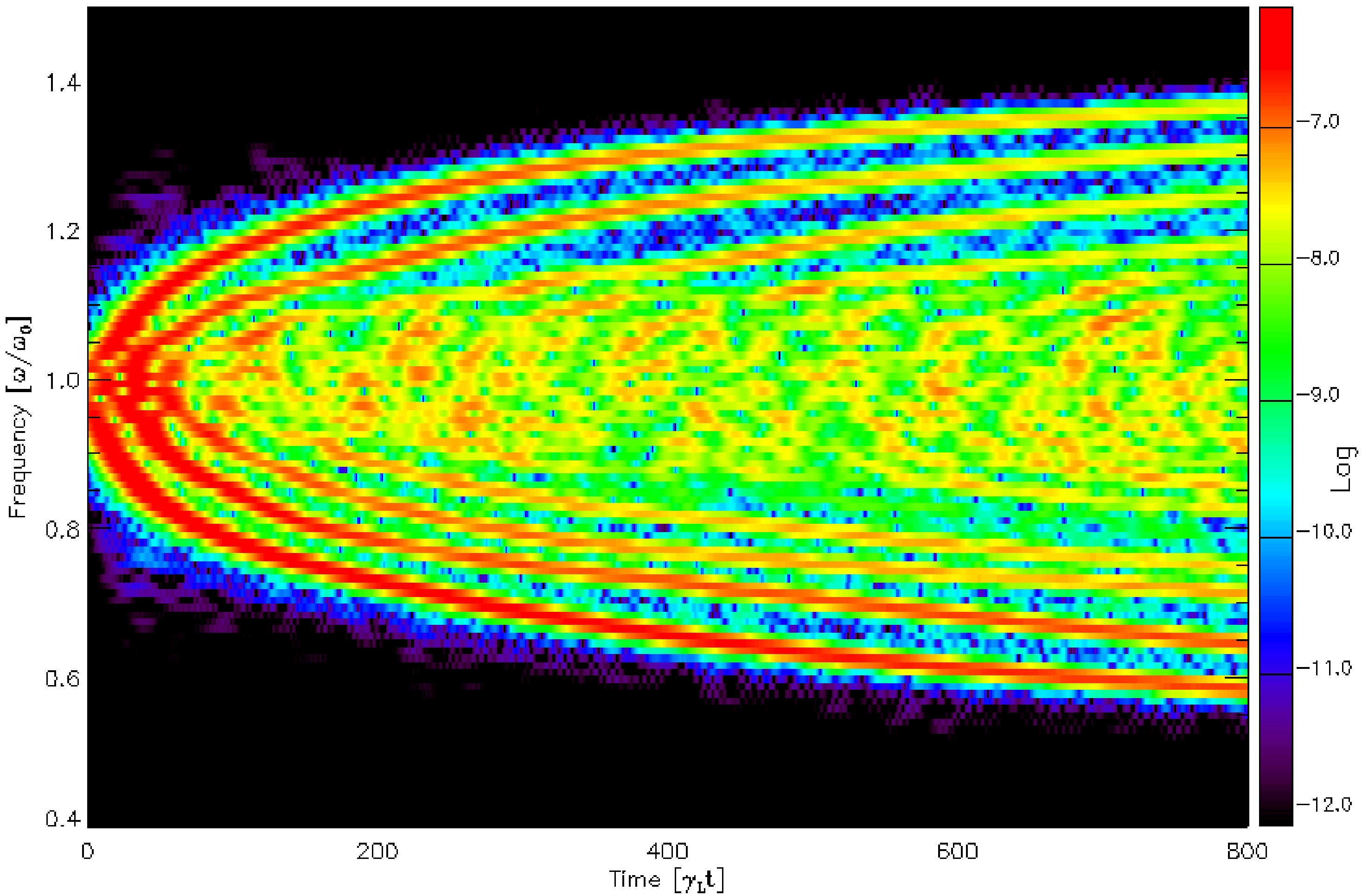
- Equilibrium:
  - $a/R_0 = 0.3$
  - $q_0 = 1.1$
  - $E_0 = 3.5$ MeV
  - $\beta_f = 3\times10^{-4}$

n = 3 TAE

Growthrate has a maximum (~6%) at ~70% of original frequency

Effect of damping

- n_p = 262,500, γ_d/ω_0 = 6%



Long term symmetric frequency sweeping, $\delta\omega \sim t^{1/2}$

HAGIS Code: Fast Particle Drag

- Introducing drag into the kinetic equation:

$$\dot{f} = \frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}} + \frac{\mathbf{F}}{m}\cdot\frac{\partial f}{\partial \mathbf{v}} = \underbrace{\nu_{\mathrm{ei}}\frac{\partial}{\partial \mathbf{v}}(\mathbf{v}f)}_{\text{Drag term, C}} + S$$

Fast ion source

- Manifests itself through a change in the characteristics of the kinetic equation (marker trajectories)

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \mathbf{x}}(\mathbf{v}f) + \frac{\partial}{\partial \mathbf{v}}(\mathcal{D}f) = 0$$

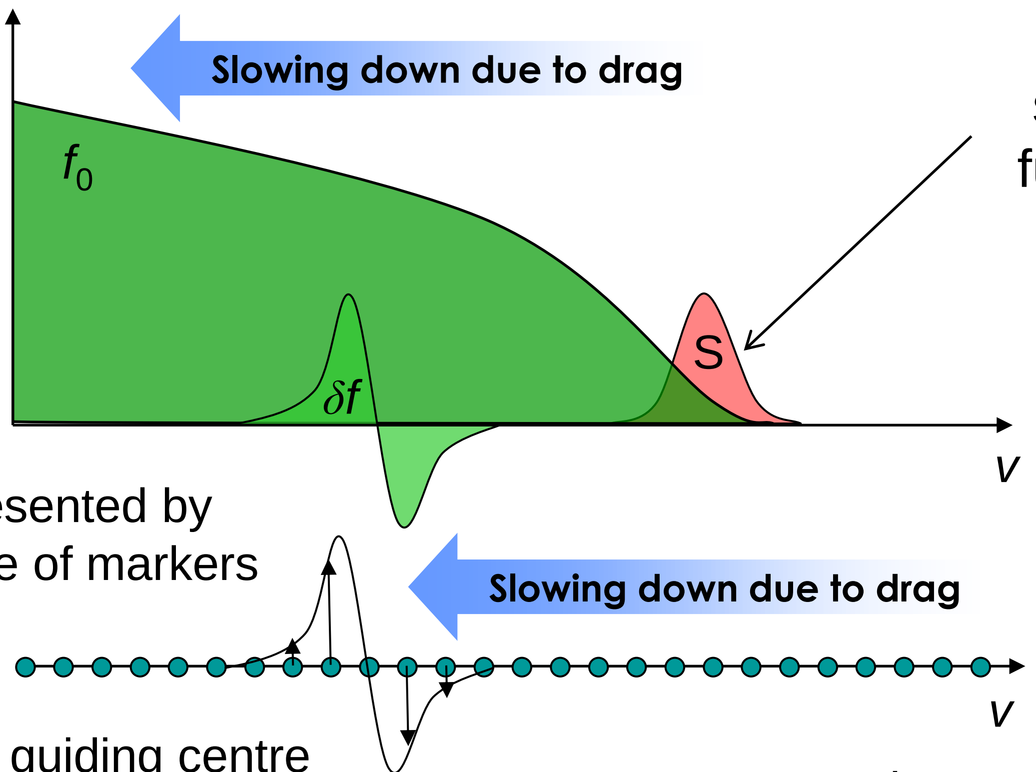
$$\frac{\partial \mathbf{v}_j}{\partial t} \longrightarrow \frac{\partial \mathbf{v}_j}{\partial t} - \nu_{\mathrm{ei}}\mathbf{v}_j$$

Model allows both *v* dependent and constant drag

HAGIS Code: Fast Particle Drag

- Including drag necessitates the inclusion of a fast ion source to maintain initial steady-state conditions

f_0: Analytic function or numerical representation



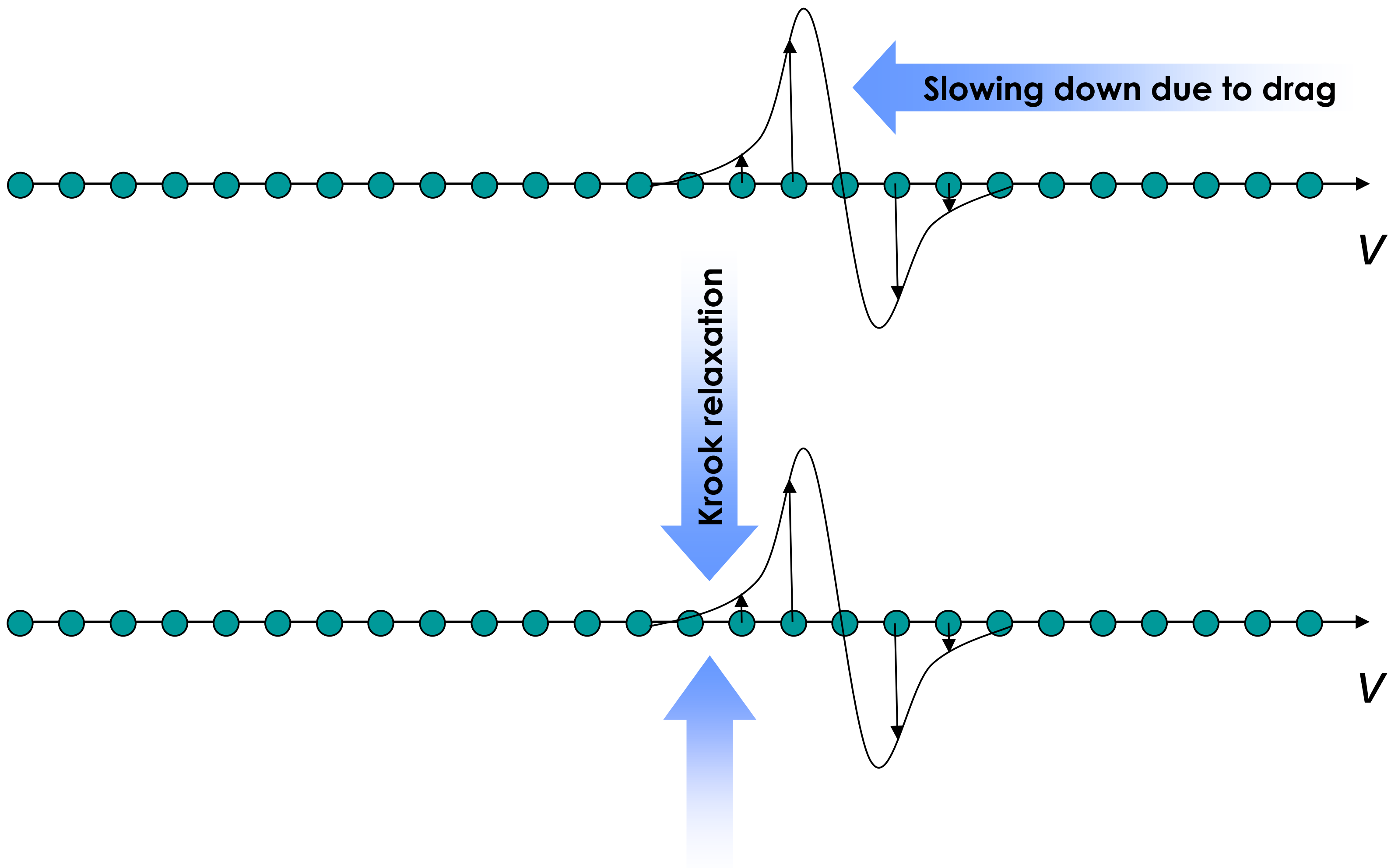
High energy source of ions: fusion alphas or beam ions

δf: Represented by ensemble of markers

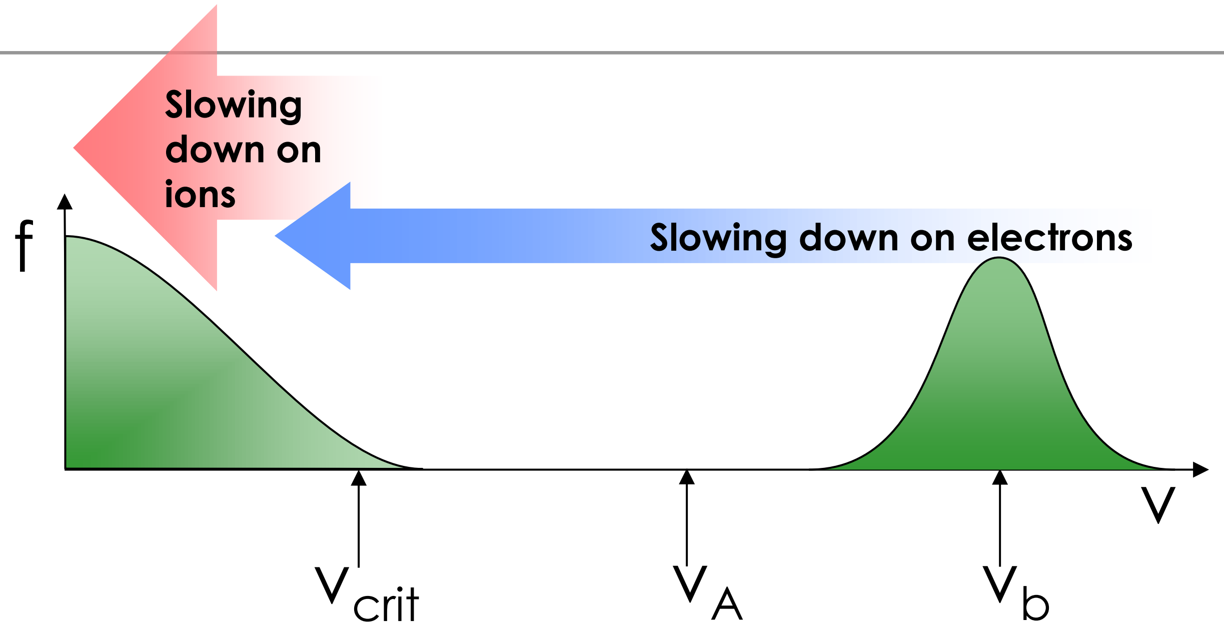
Markers follow guiding centre trajectories and slow down. They may be removed from simulation at low v

Marker source is handled by loading markers to high *v*

Perturbation to distribution moves through phase space affecting gradients and stability

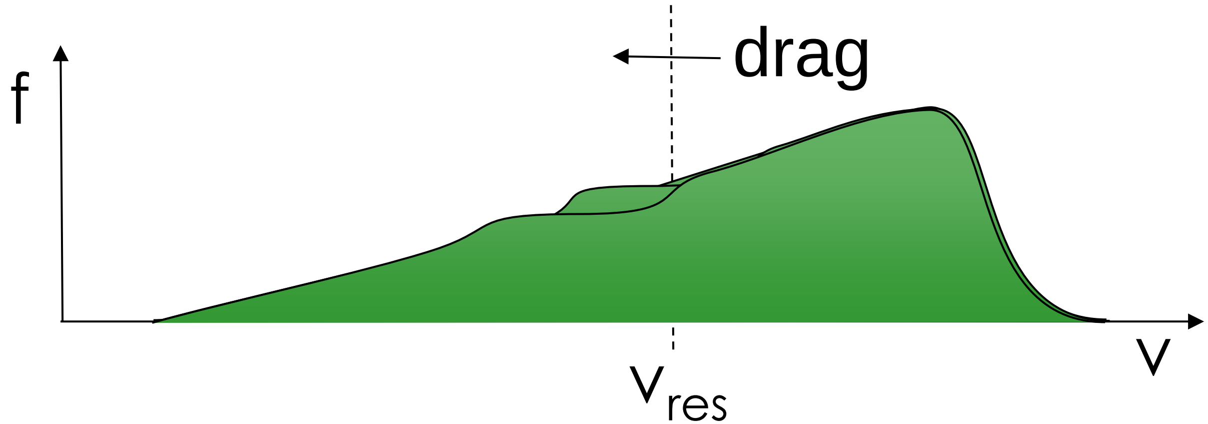


Super-Alfvénic ion source and effect of drag



NBI blip or alpha source for Δt

Bump-on-tail distribution

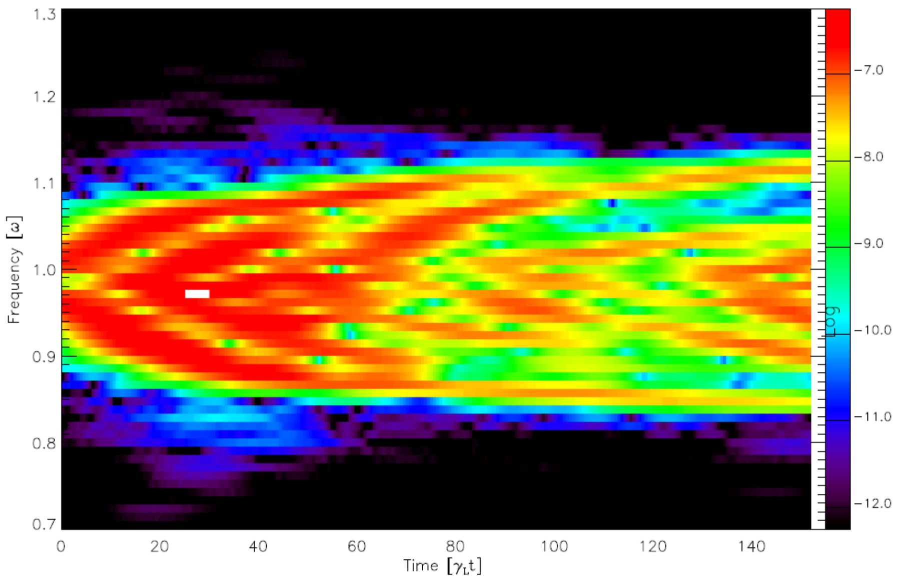


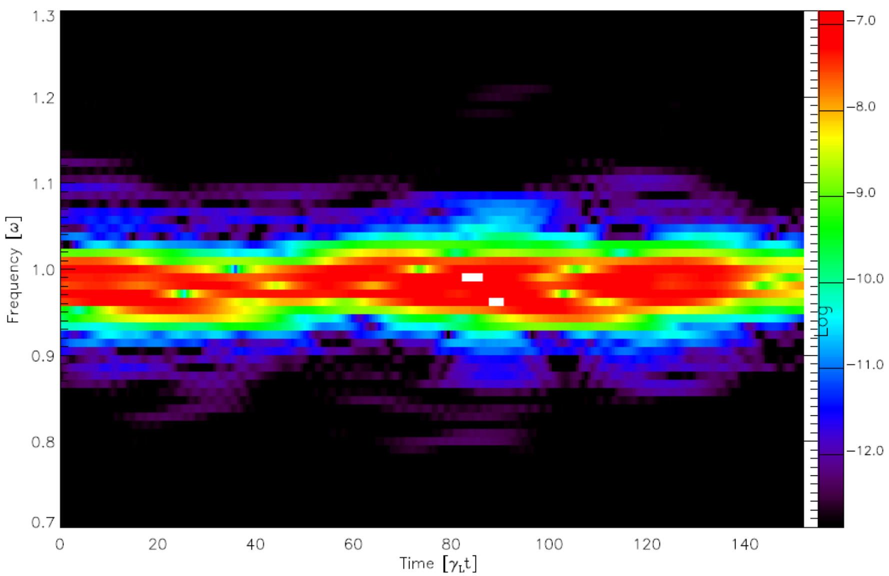
Wave flattens distribution and removes drive

Drag moves flat spot leading to increased drive and explosive growth!

Effect of (Krook) relaxation

- If ν_{eff} is ~1% of γ_L then frequency sweeping structures are destroyed after ~100 $\gamma_L t$

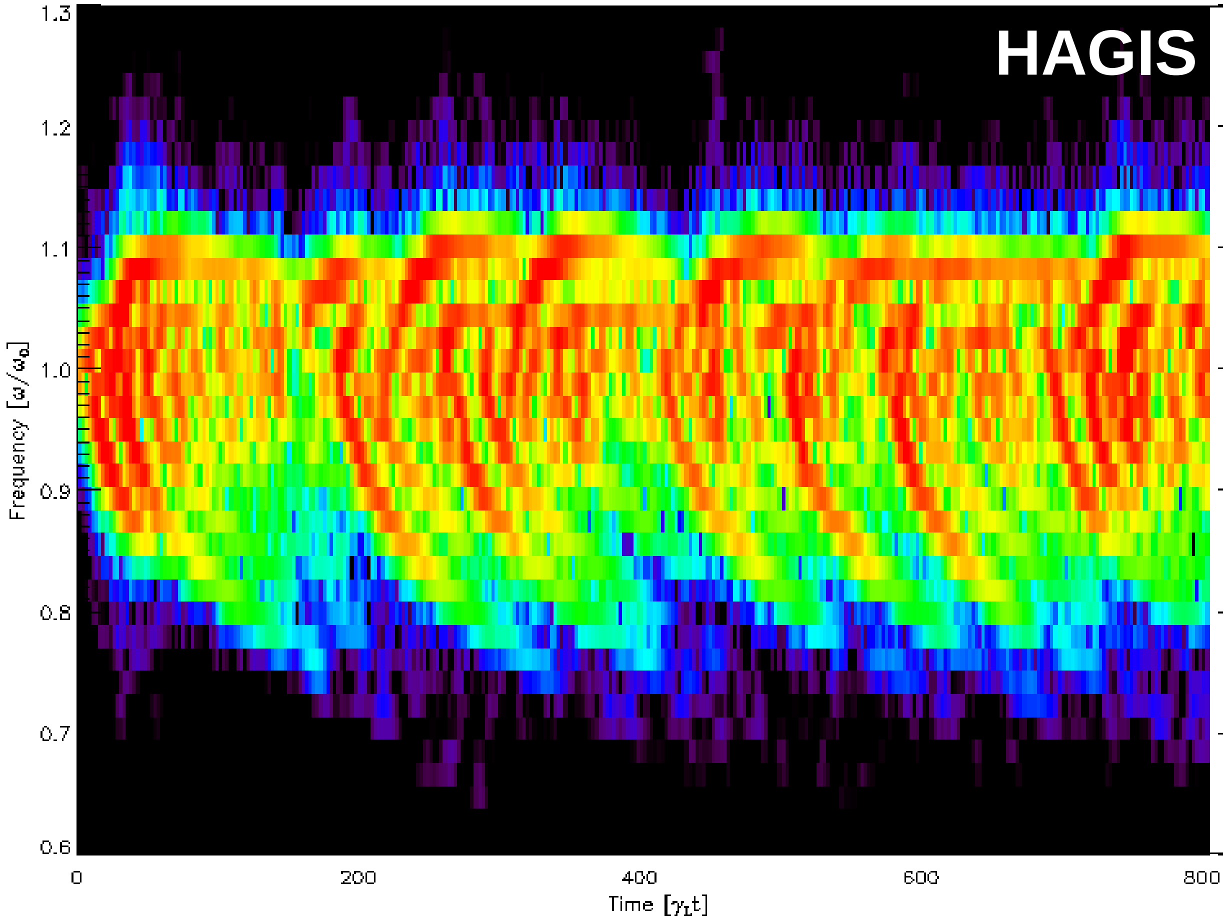


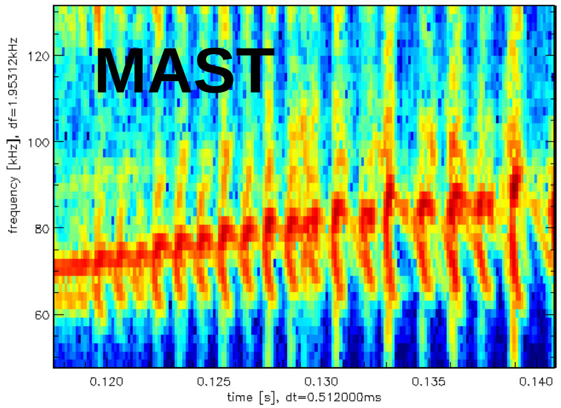


- Increasing Krook relaxation to 10% almost completely eradicates any mode sweeping

Nonlinear Behaviour: Drag + Krook

- n_p = 262,500, γ_L/ω_0 = 6.12%, γ_d/ω_0 = 6%, ν_{ei}/ω_0 = 0.3%, ν_{eff}/ω_0 = 1%



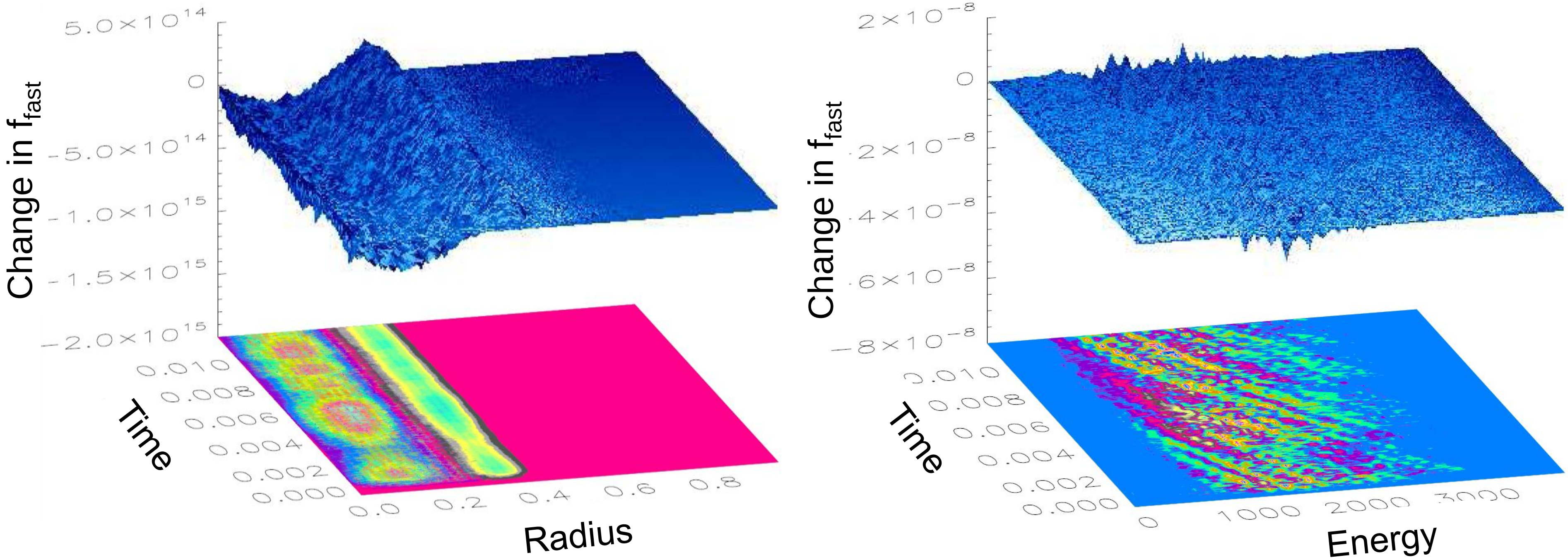


Frequency sweeping TAE in MAST #22807

- Asymmetric, repetitive, frequency sweeps: $\delta\omega/\omega_0 \sim \pm$ 30%

Fast Ion Redistribution: Drag + Krook

- Changes to fast ion distribution due to nonlinear self-consistent wave-particle interaction:
  - Extensive and sustained redistribution

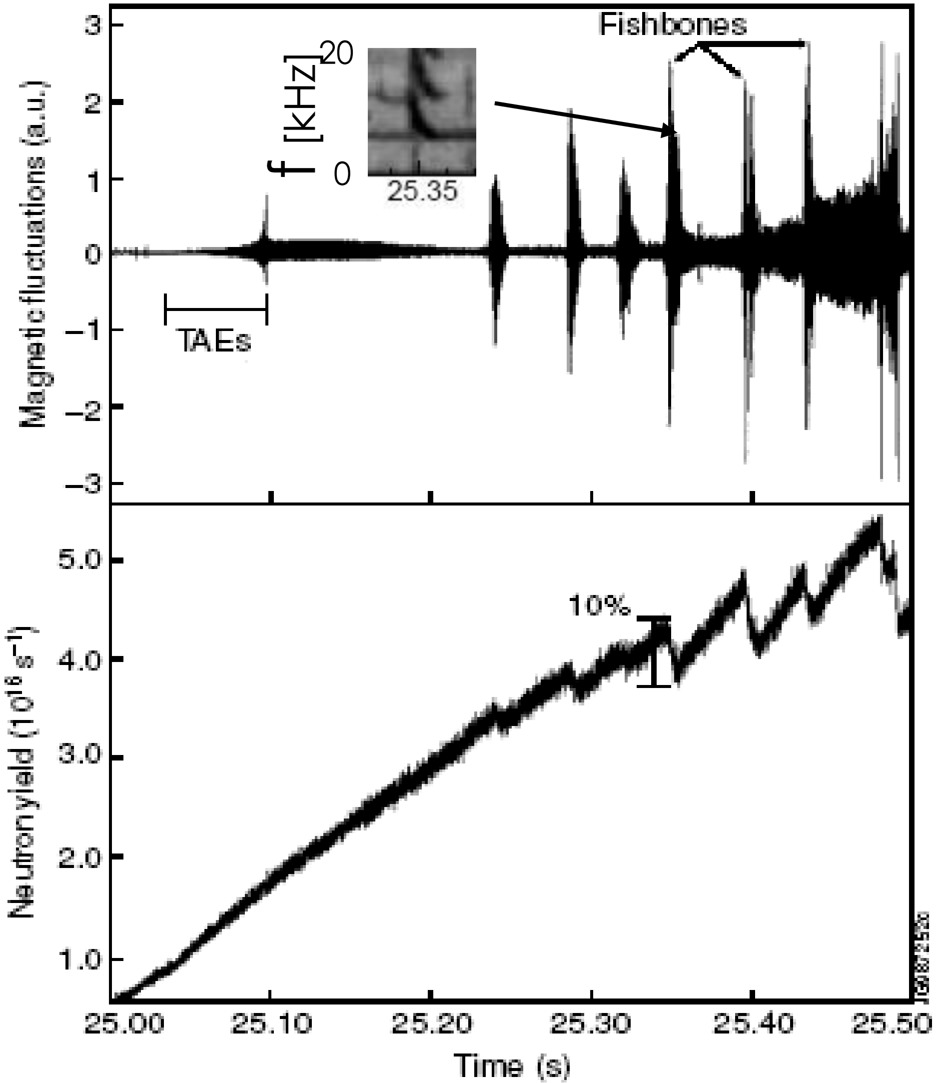


- n_p = 262,500, γ_L/ω_0 = 6.12%, γ_d/ω_0 = 6%, ν_{ei}/ω_0 = 0.3%, ν_{eff}/ω_0 = 1%

FISHBONES

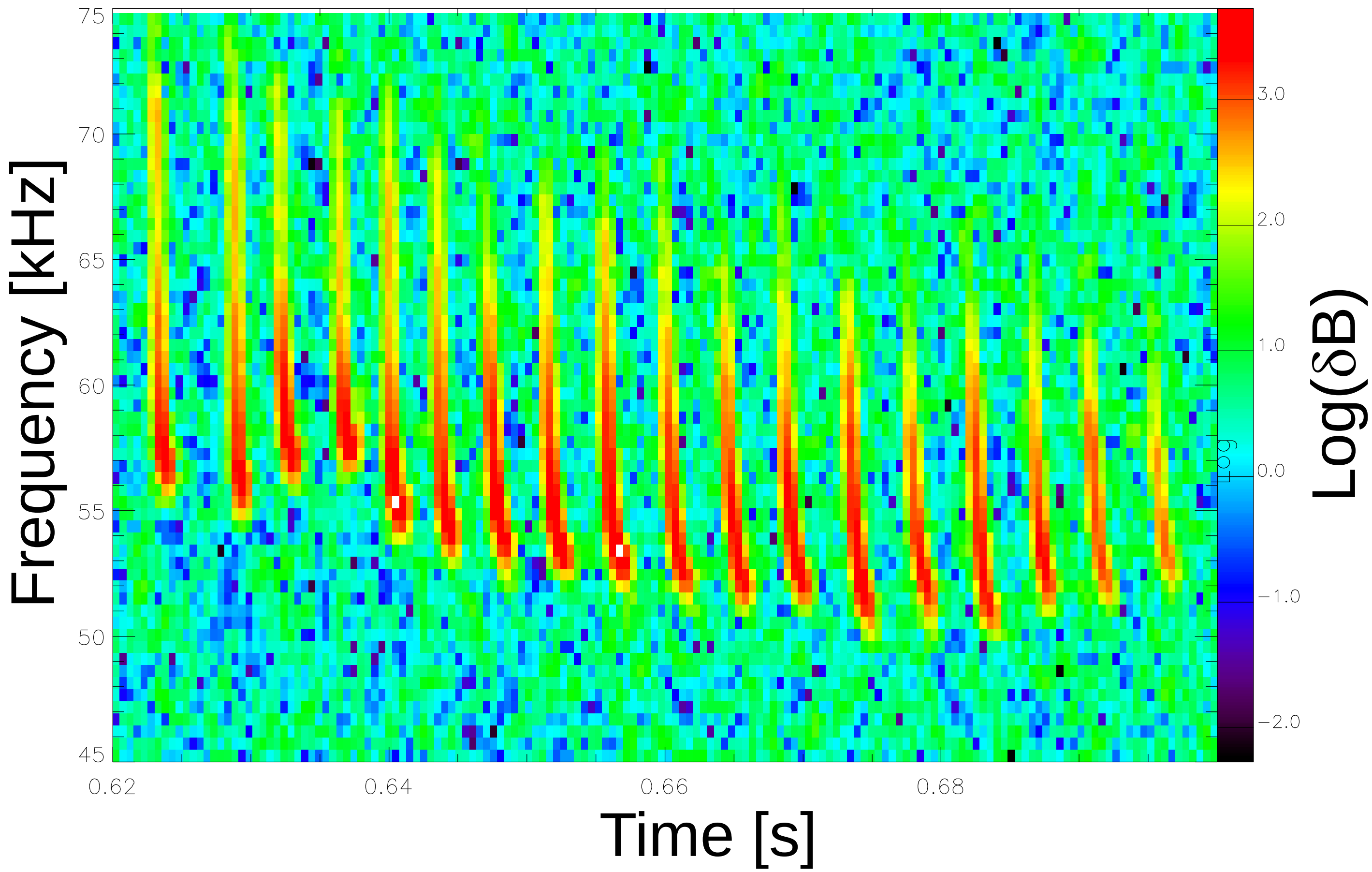
Fast Particle Losses in JET

- NBI heating
  - $v_b \sim v_A$
- 10% drop in neutron yield due to ‘fishbones’



D.N. Borba *et al*., Nucl. Fusion **40** (2000)

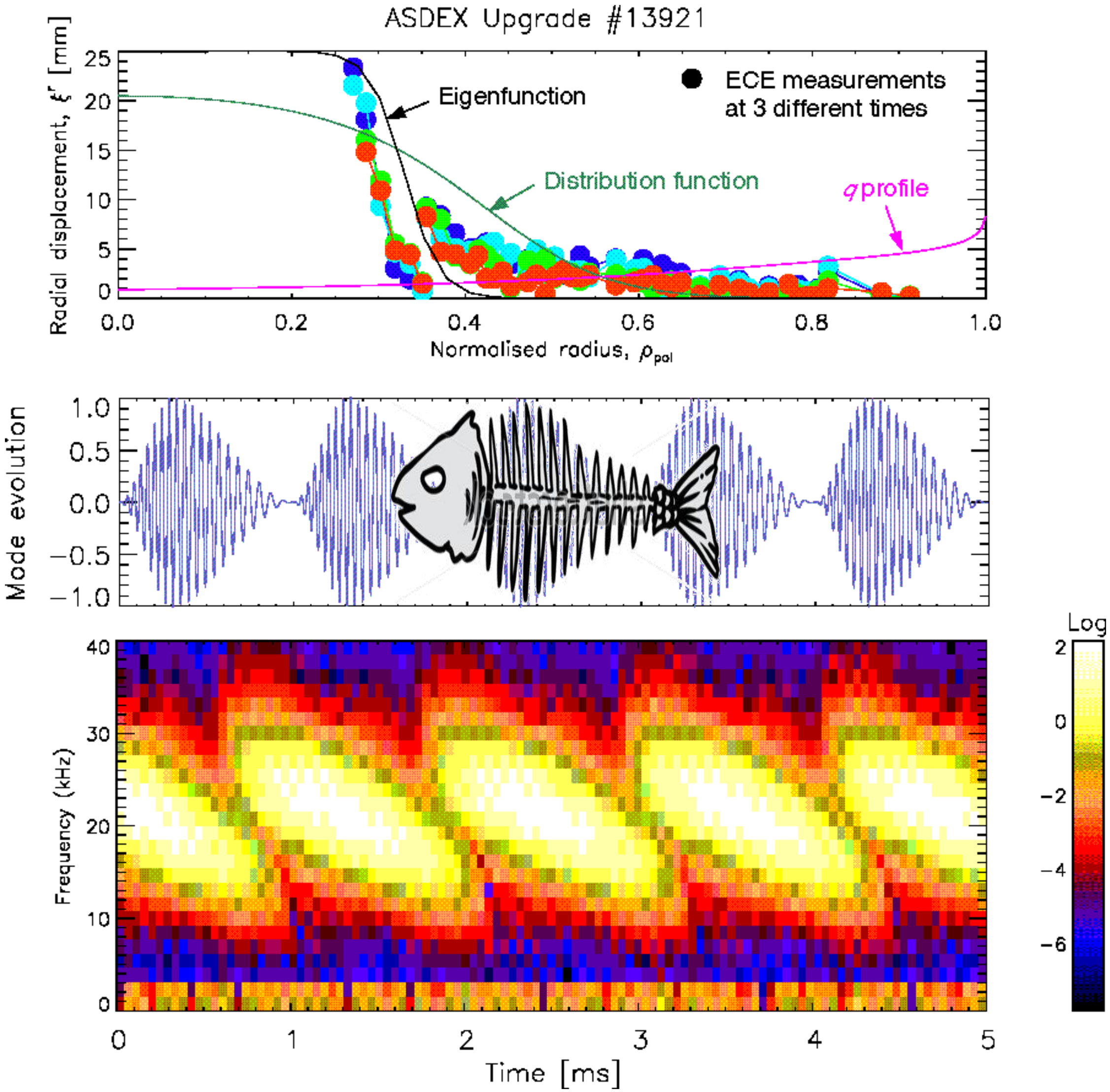
Fishbone Instability



- Frequency sweeping mode driven by fast particles
- Consistent MHD/kinetic description being developed

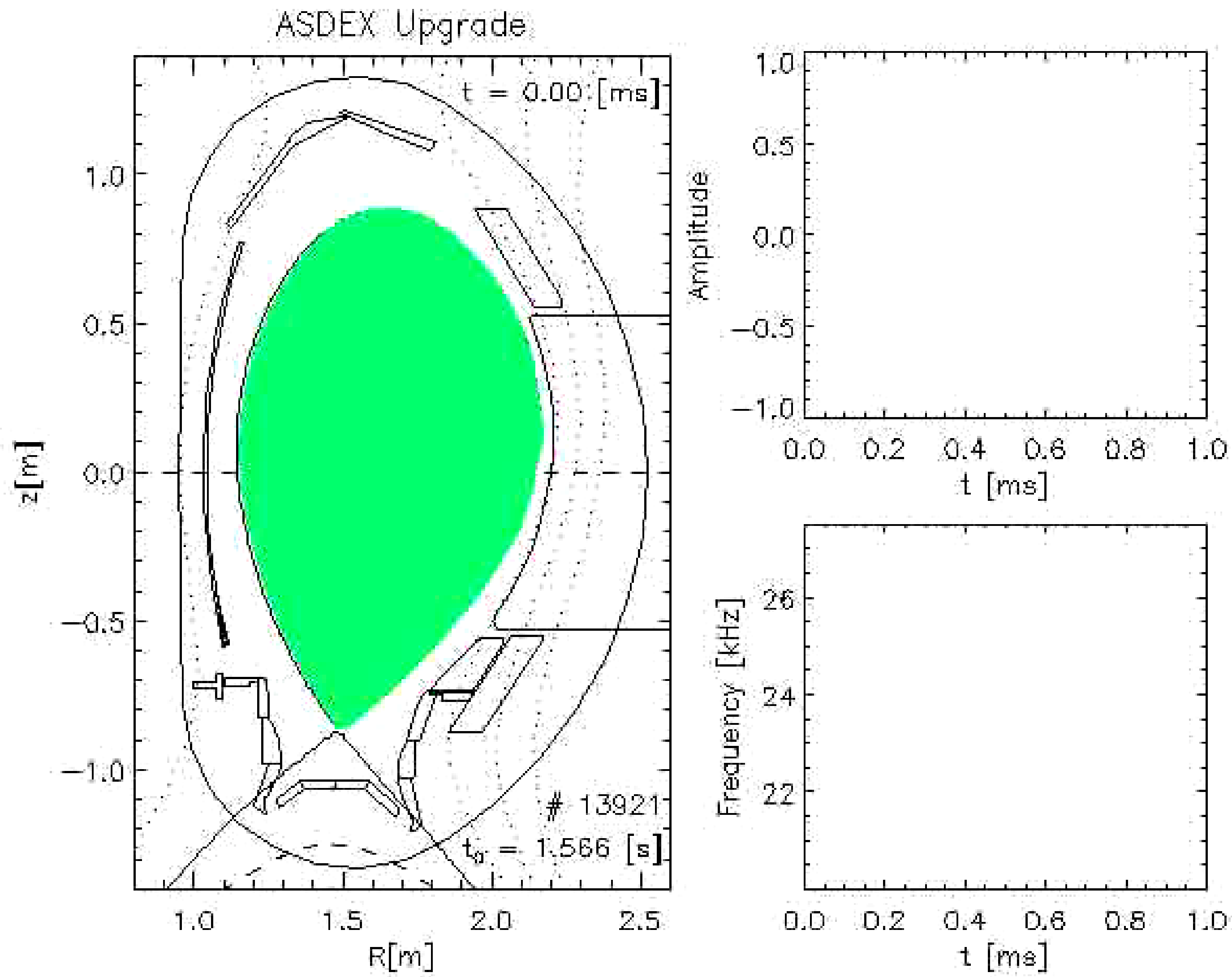
A. Ödblom *et al.*, Phys. Plasmas **9** (2002) 155

Modelling Fishbones in ASDEX Upgrade

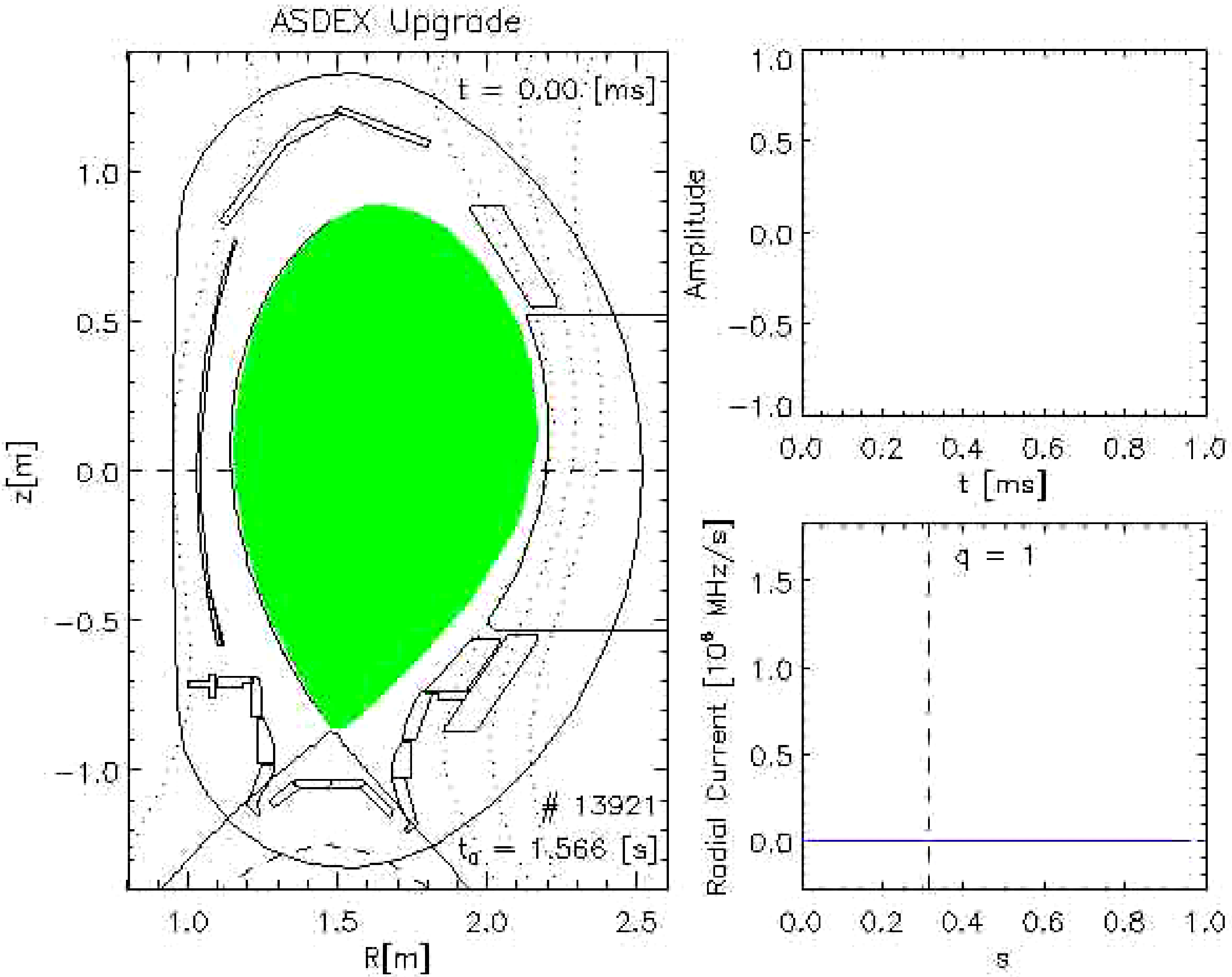


- m=1,n=1 internal kink
- Linear frequency chirps (27.5→20 kHz)
- Repetition rate: 1ms
- Slowing down distribution of 60 keV NBI ions
- $\langle\beta_{fast}\rangle = 0.36\%$

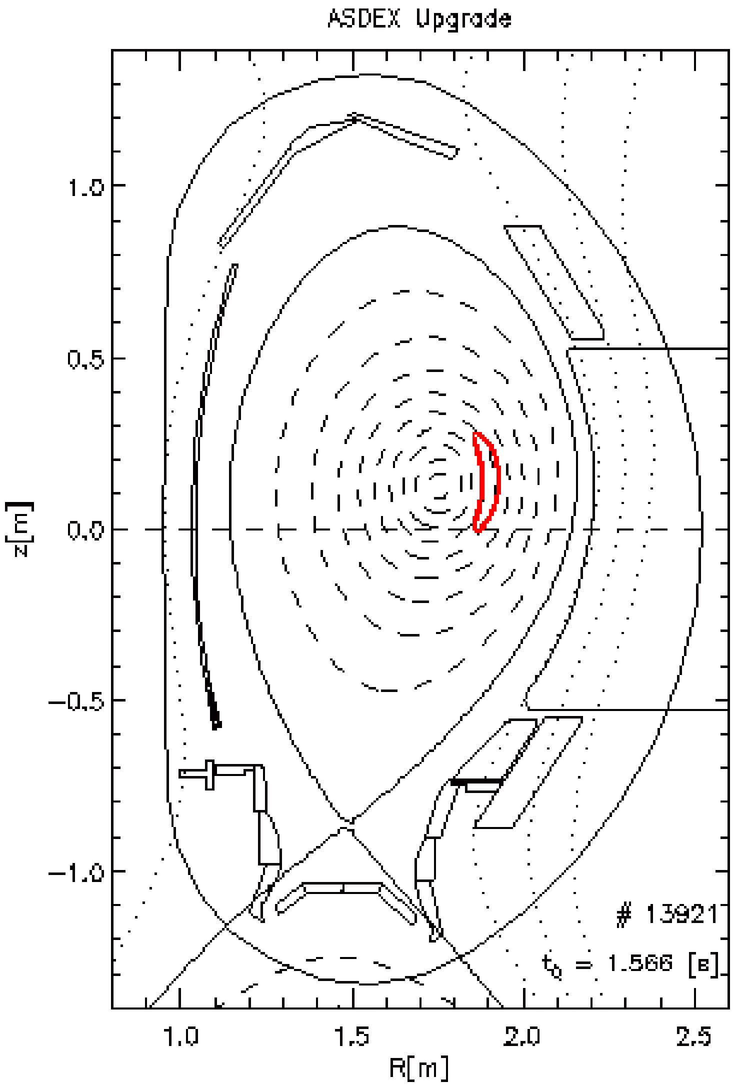
Fishbone Evolution



Fishbone Simulation



Current Carrying Ion

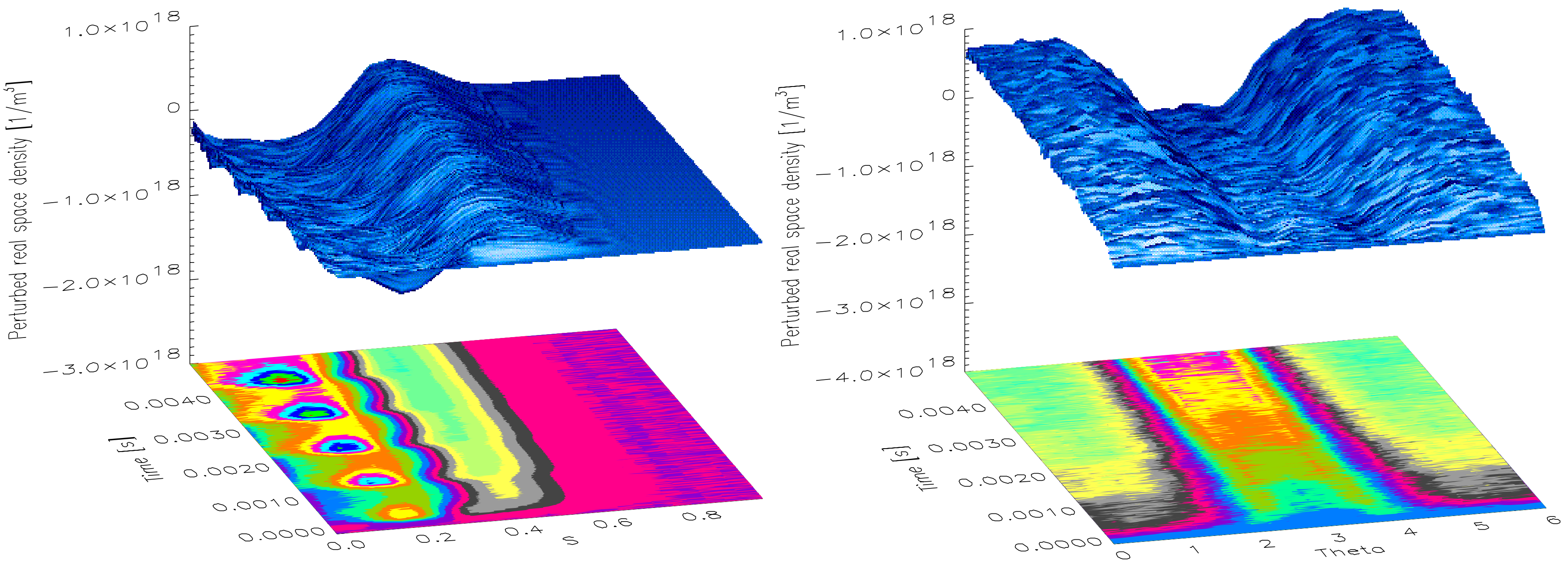


- Trapped ion at $q = 1$ surface
- Energy, E = 55 keV
- Precession frequency, ω_ϕ = 7 kHz
- Bounce frequency, ω_b = 41 kHz

china eu india japan korea russia usa

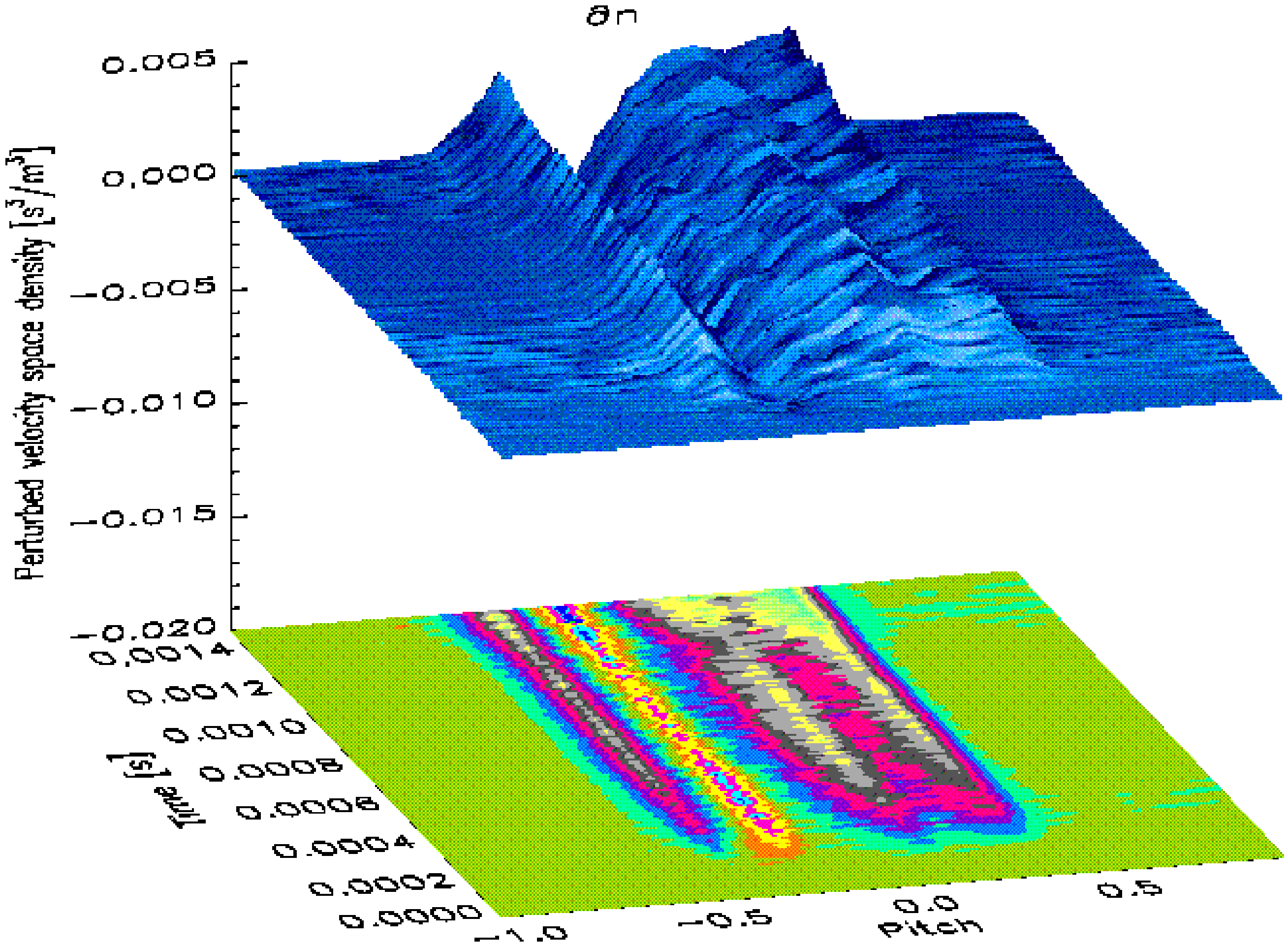
Spatial redistribution due to fishbones

- Fast ions radially expelled towards low field side



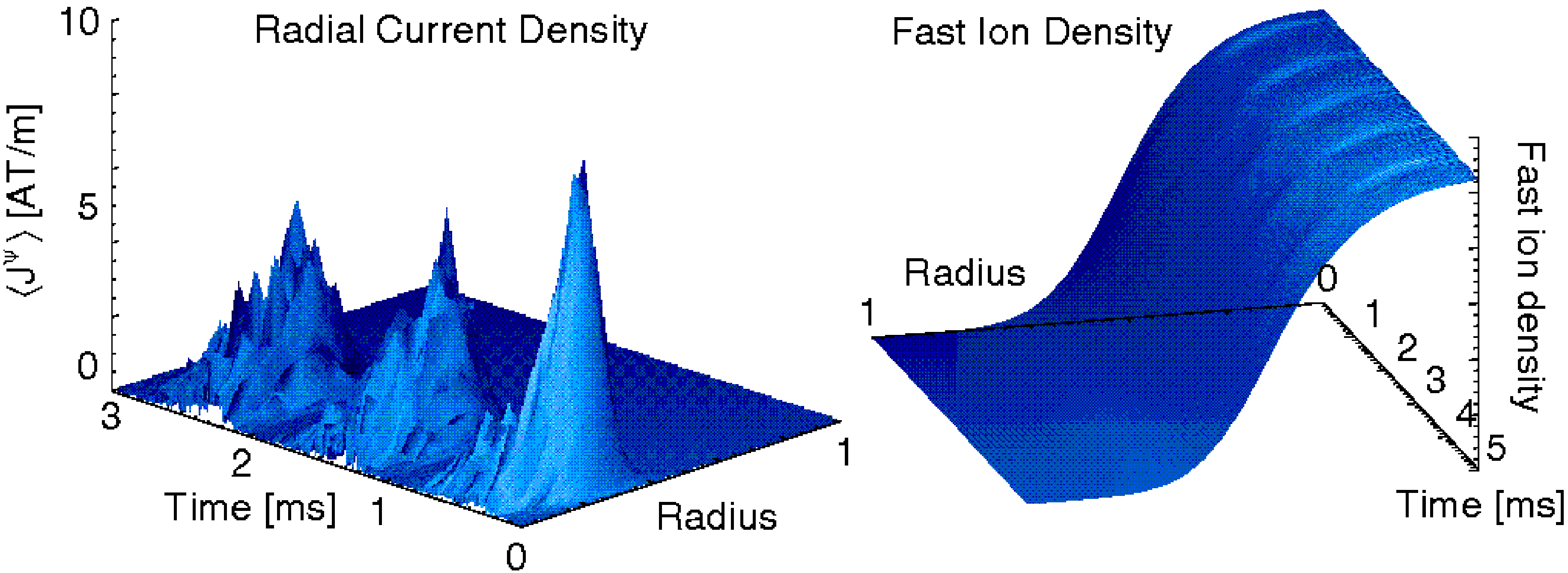
Pitch Angle Redistribution

- Change in trapped/passing fast ion distribution



Fast Ion Radial Current

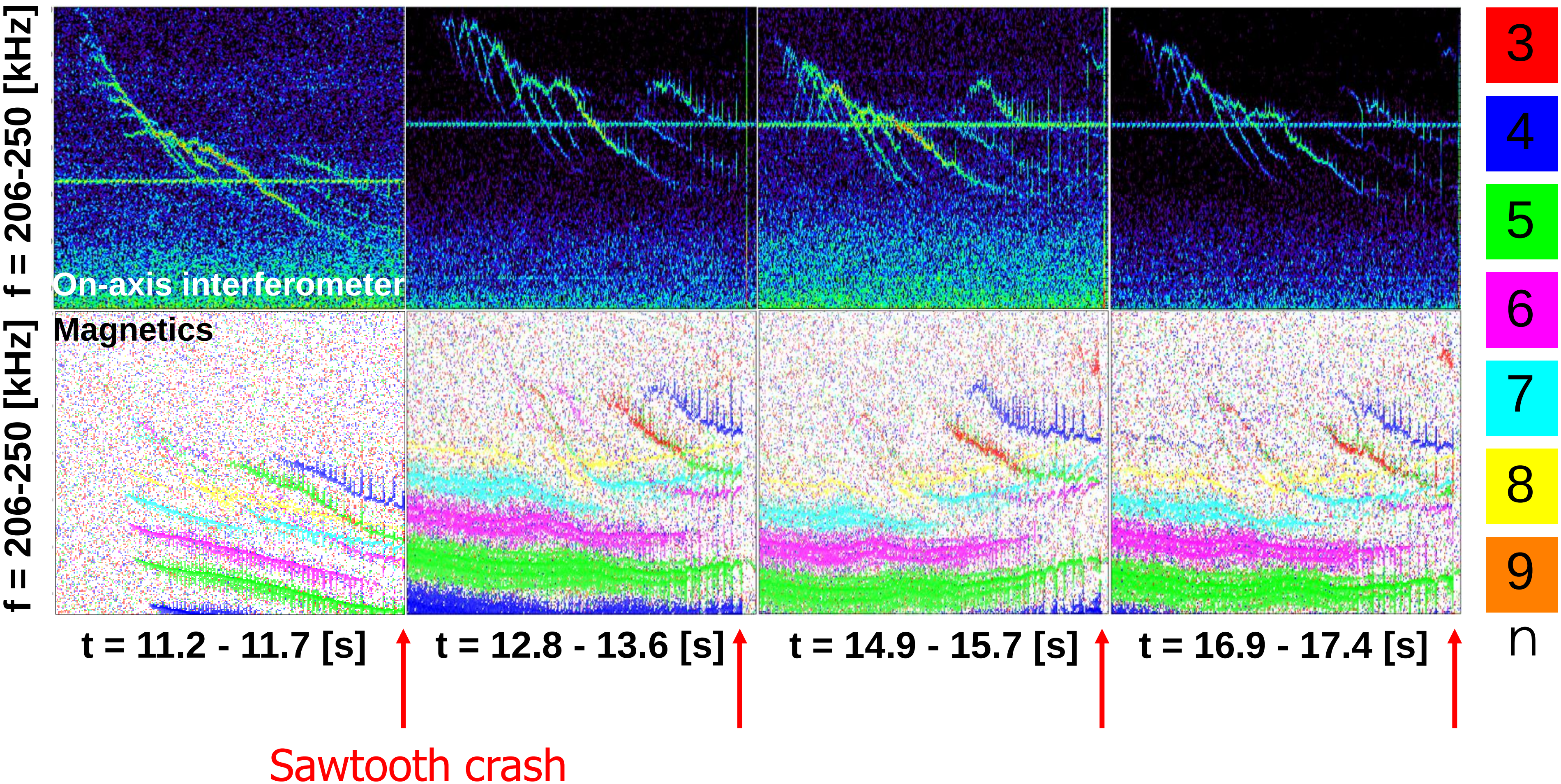
- δf simulation with HAGIS code gives $<J^{\psi}(t)>$ and variation of fast ion distribution function



FAST ION LOSSES DUE TO TORNADO MODES IN JET

Tornado modes in JET

- *Every* “monster” sawtooth crash preceded by tornado modes

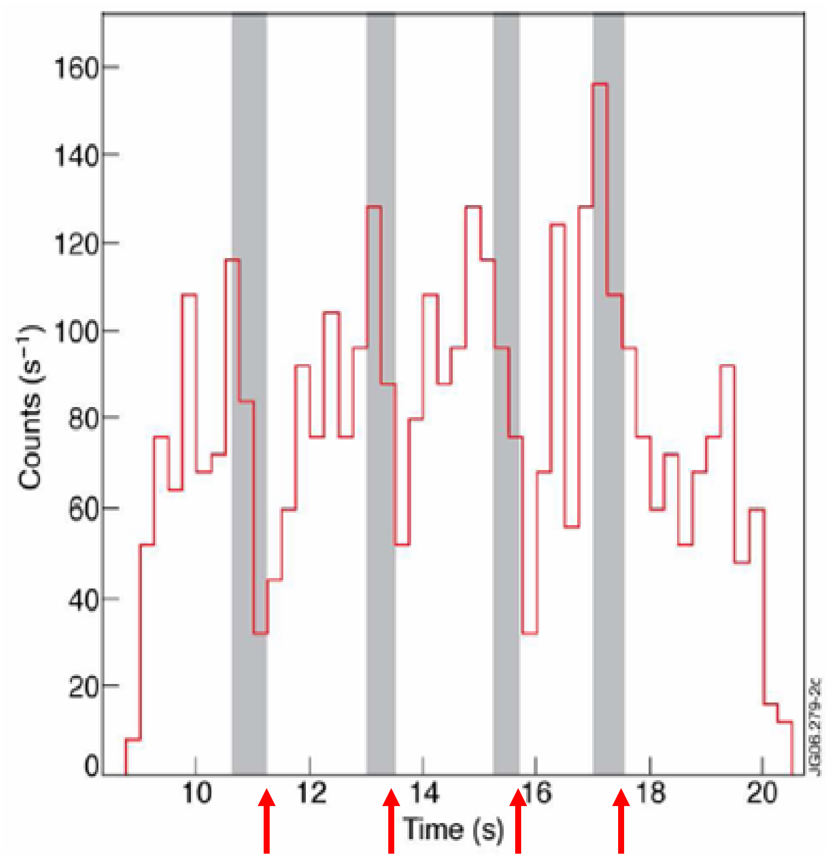


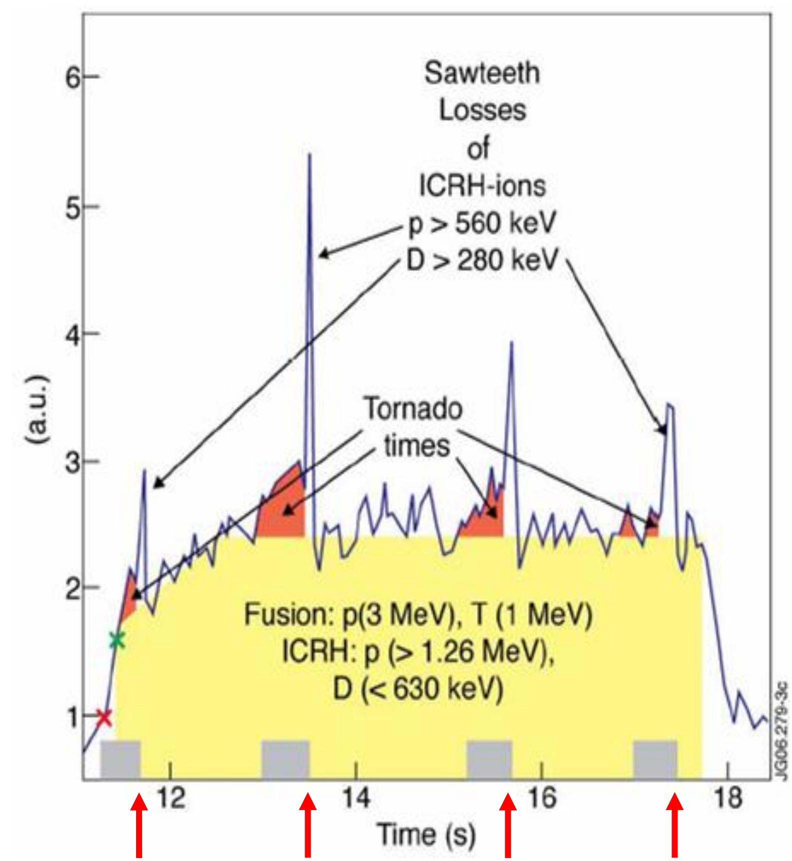
Observations of Fast Ion Losses in JET

Loss measurements increase during tornado mode activity

3.1-MeV γ-ray emission from $^{12}C(d,p\gamma)^{13}C$;
Deuterons with E>500 keV

Scintillator probe



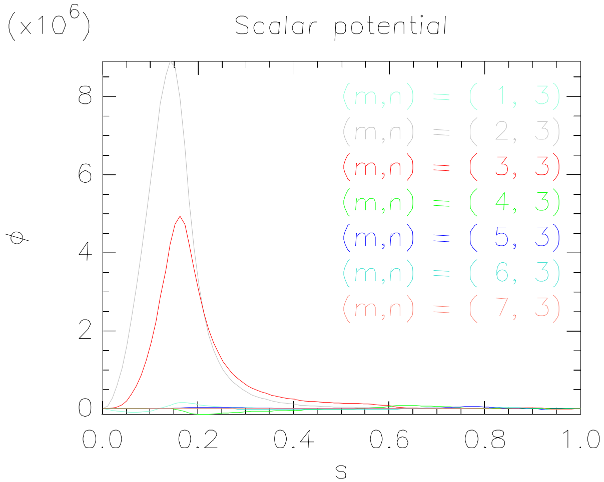


Sawtooth crashes

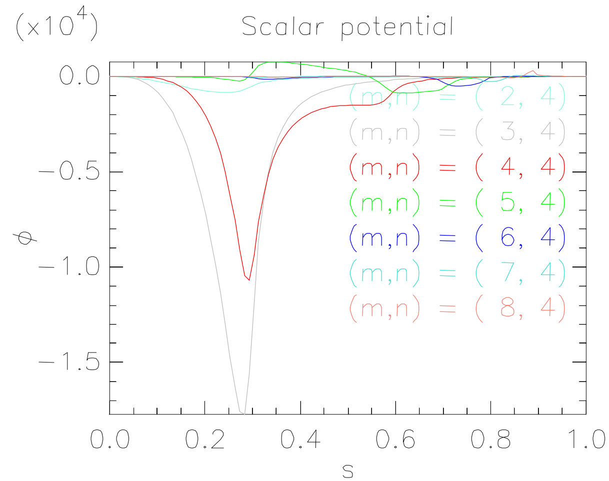
Sawtooth crashes

V Kiptily

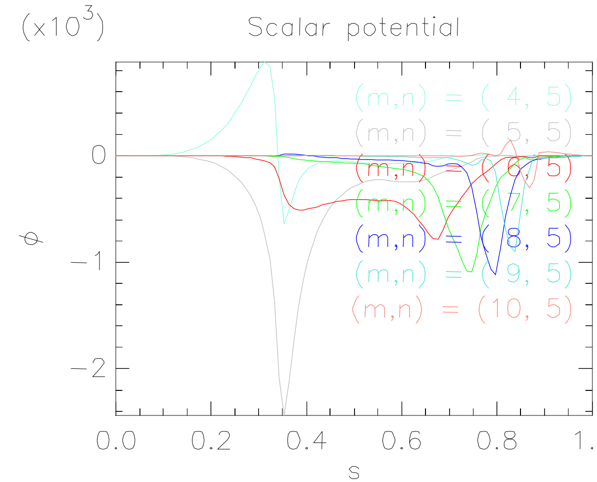
TAE Mode Structure



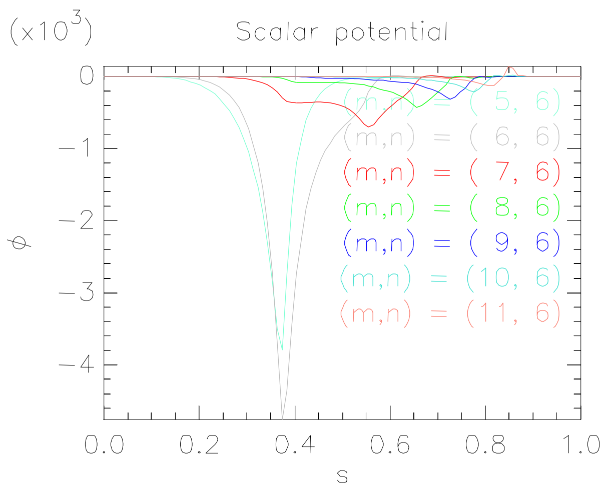
n = 3



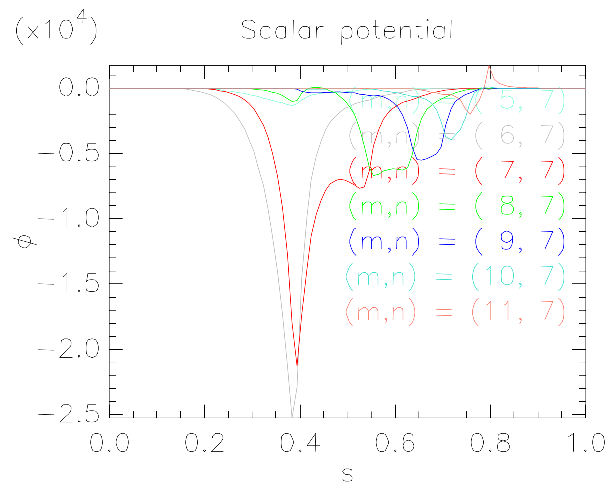
n = 4



n = 5



n = 6

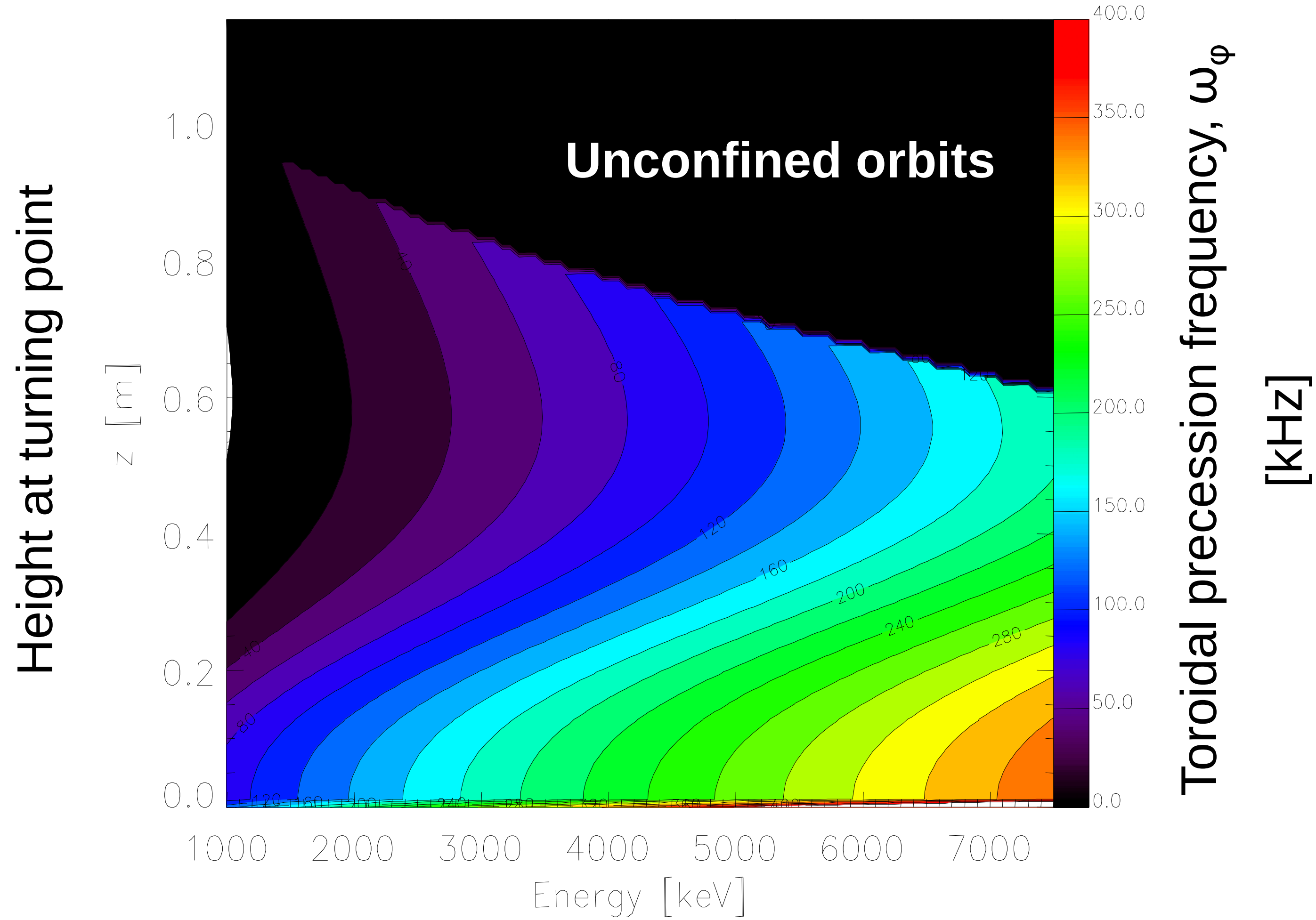


n = 7

- Linear MHD eigenfunctions calculated with CASTOR code
  - Equilibrium from HELENA code

Fast Ion Properties

- Determine natural particle frequencies, ω_φ and ω_θ



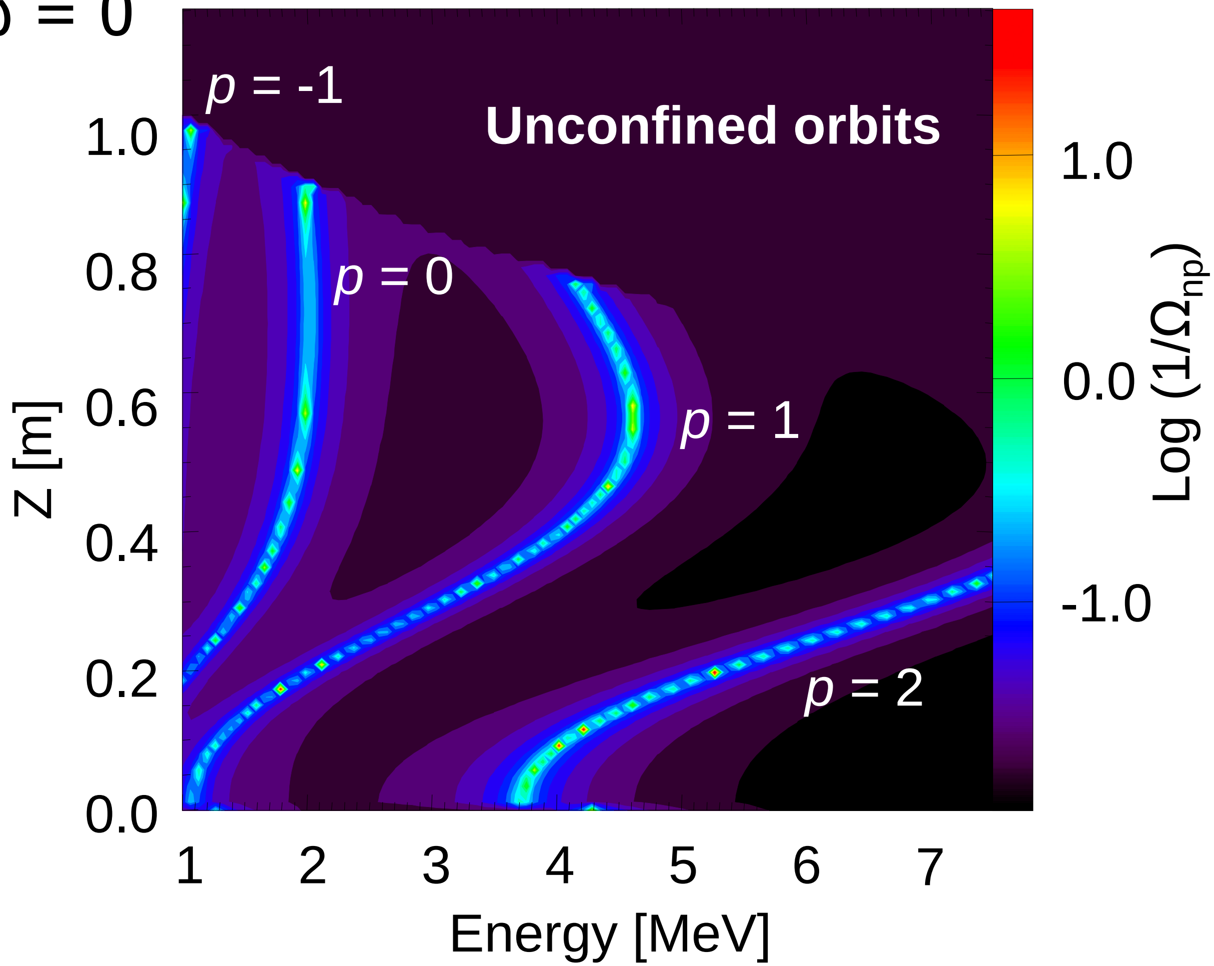
Resonant ICRH ions

Resonance condition:

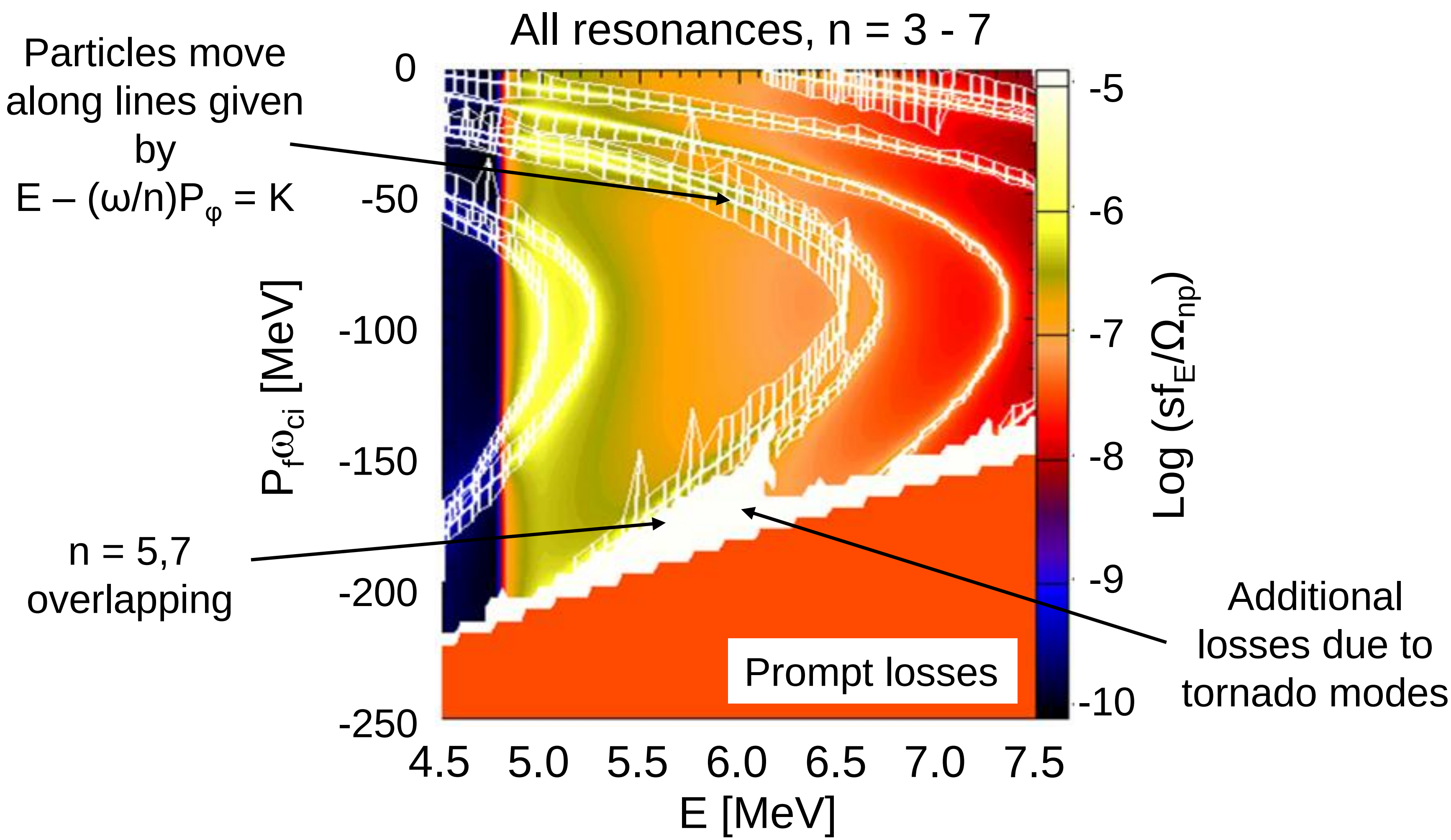
- $\Omega_{np} = n\,\omega_\varphi - p\,\omega_\theta - \omega = 0$

n = 3 tornado mode:

- p = -1 → 2
- f = 283 kHz



Resonance Overlap



- Overlap between resonances explains observed loss

Summary

- Physics of fast ion driven instabilities well understood
- Fast particles drive instabilities and are in turn re-distributed and, in some cases, lost
  - Consistent *nonlinear* story emerging
- Nonlinear modelling of fast ion driven instabilities
  - Multiple modes interacting through driving fast ion distribution
  - Determination of amplitude of frequency sweeping modes in MAST
  - Radial fast ion current due to fishbones in ASDEX Upgrade
  - Fast ion losses due to tornado modes in JET
- Models start to successfully describe rich nonlinear phenomena near marginal stability
  - Mode saturation, pitchfork splitting and frequency sweeping
- Fast particle driven modes remain a valuable diagnostic tool
  - MHD spectroscopy ($q_{min}(t)$ from Alfvén cascades)