Electromagnetic gyrokinetic simulation of turbulence in torus plasmas

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Outline

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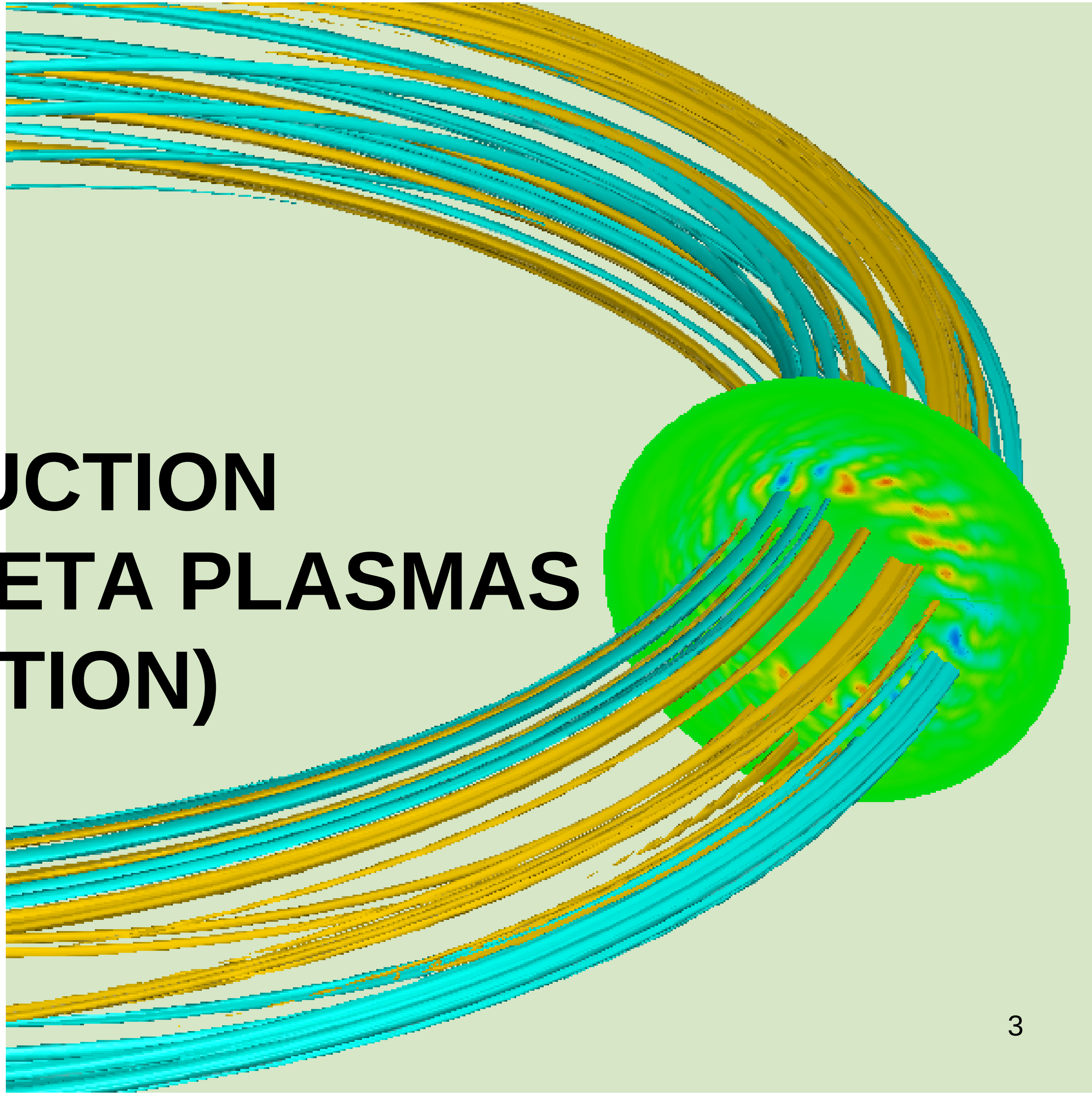
  - Beta dependence of turbulent transport
  - Saturation problem of turbulence at finite-beta
- Summary

INTRODUCTION
FINITE BETA PLASMAS
(MOTIVATION)



Motivation of EMGK analysis

- Electromagnetic gyrokinetic simulation enables us to study turbulent transport in finite beta torus plasmas.
- Beta dependence of anomalous transport.
  - Fusion reaction rate
  - Fraction of bootstrap current

$$\beta = \frac{4\pi P}{B^2/2} \qquad \beta_i = \frac{4\pi n_0 T_i}{B^2/2}$$

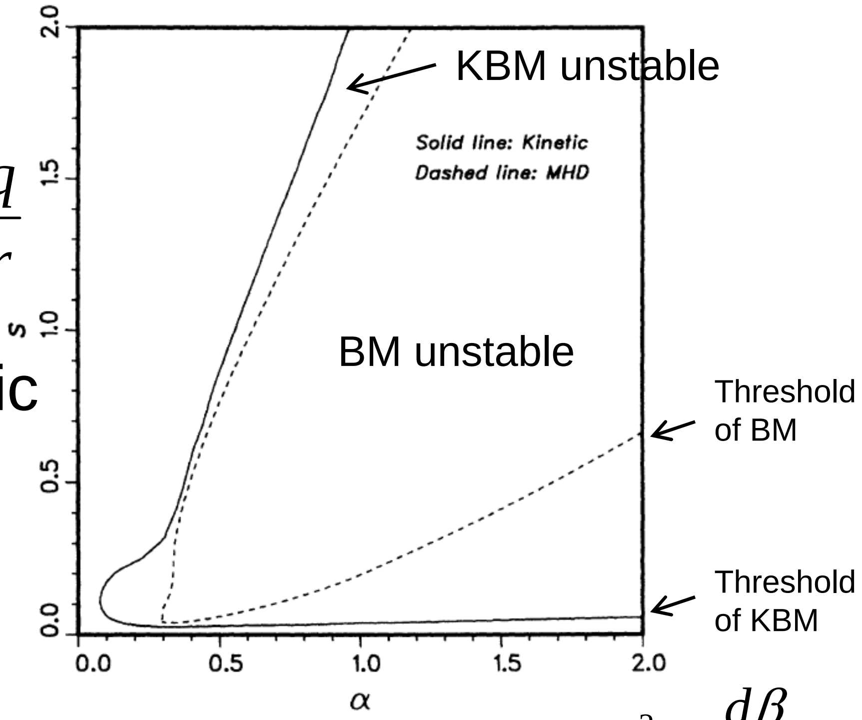
A typical EM instability

- Ballooning modes
  - related to the edge localized mode.
- Kinetic ballooning modes
  - Appears in BM stable region

Core density collapses in LHD experiments appear when BM is unstable.

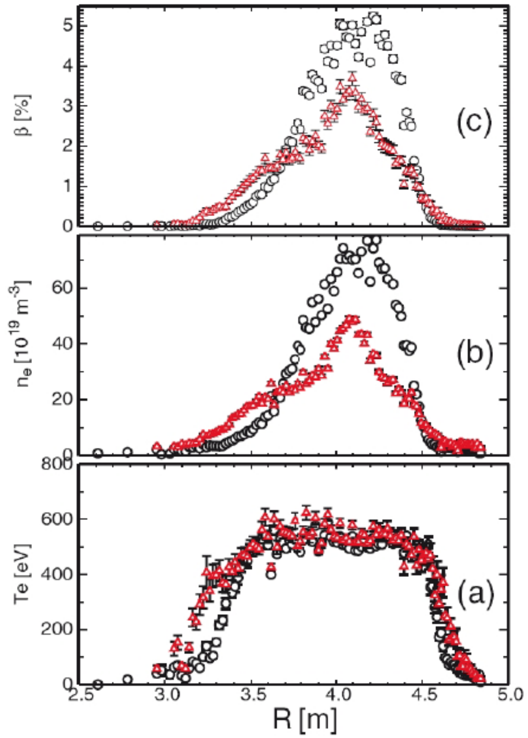
$$s = \frac{r}{q}\frac{dq}{dr}$$

Magnetic shear



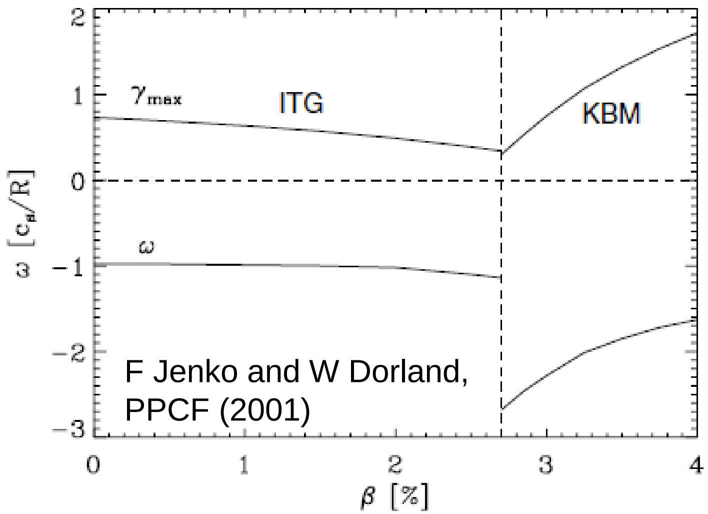
Pressure gradient $\alpha = -q^2 R_0 \frac{d\beta}{dr}$

A. Hirose, Phys. Rev. Lett., 3993 (1994)



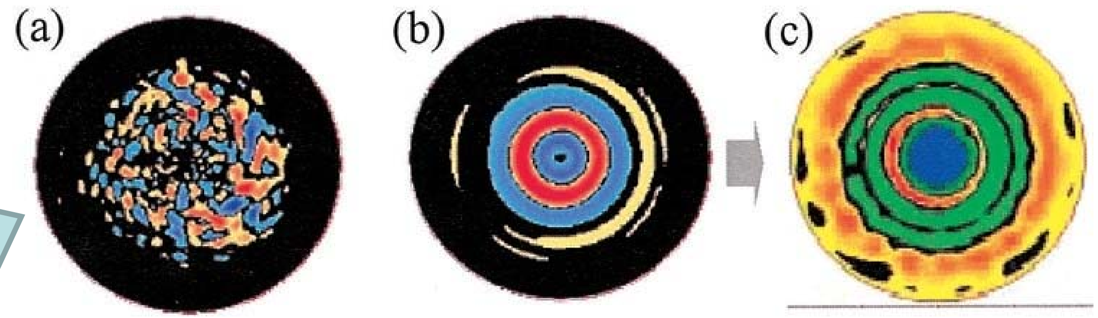
Ohdachi, Contrib. Plasma Phys. 2010

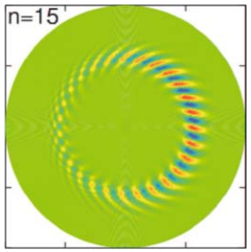
Zonal structure formation affects saturation of instabilities



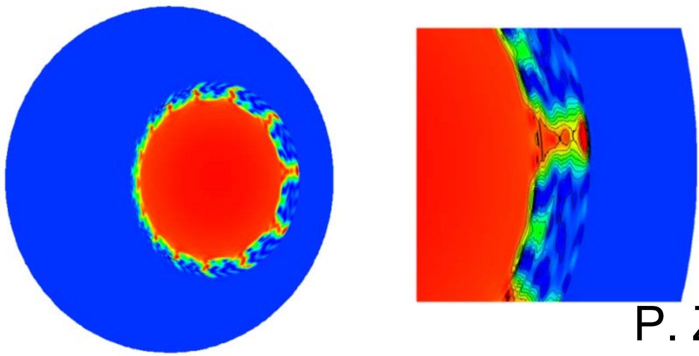
- Low beta: Ion temperature gradient mode (ITG)
  - Stabilized by zonal flow structure.

Y. Kishimoto





- High beta: Ballooning (MHD) instability
  Acceleration by finger-like structure



P. Zhu

Mode structures of ITG and KBM are similar. Both of them have ballooning structure in the linear growth.

Electromagnetic gyrokinetic equations

Numerical difficulty in EMGK

ELECTROMAGNETIC GYROKINETIC EQUATIONS

Assumptions in GK

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\right] F_s = C_s(F_s)$$

- Spatial structure: Flute approximation
- Time scale: Drift-ordering
- Amplitude: Small

$$\frac{\delta f_s}{F_{Ms}} \approx \frac{k_{//}}{k_\perp} \approx \frac{\omega}{\Omega_s} \approx \frac{e\delta\phi}{T_s} \approx \frac{\delta A_{//}}{B_0\rho_s} \approx \varepsilon$$

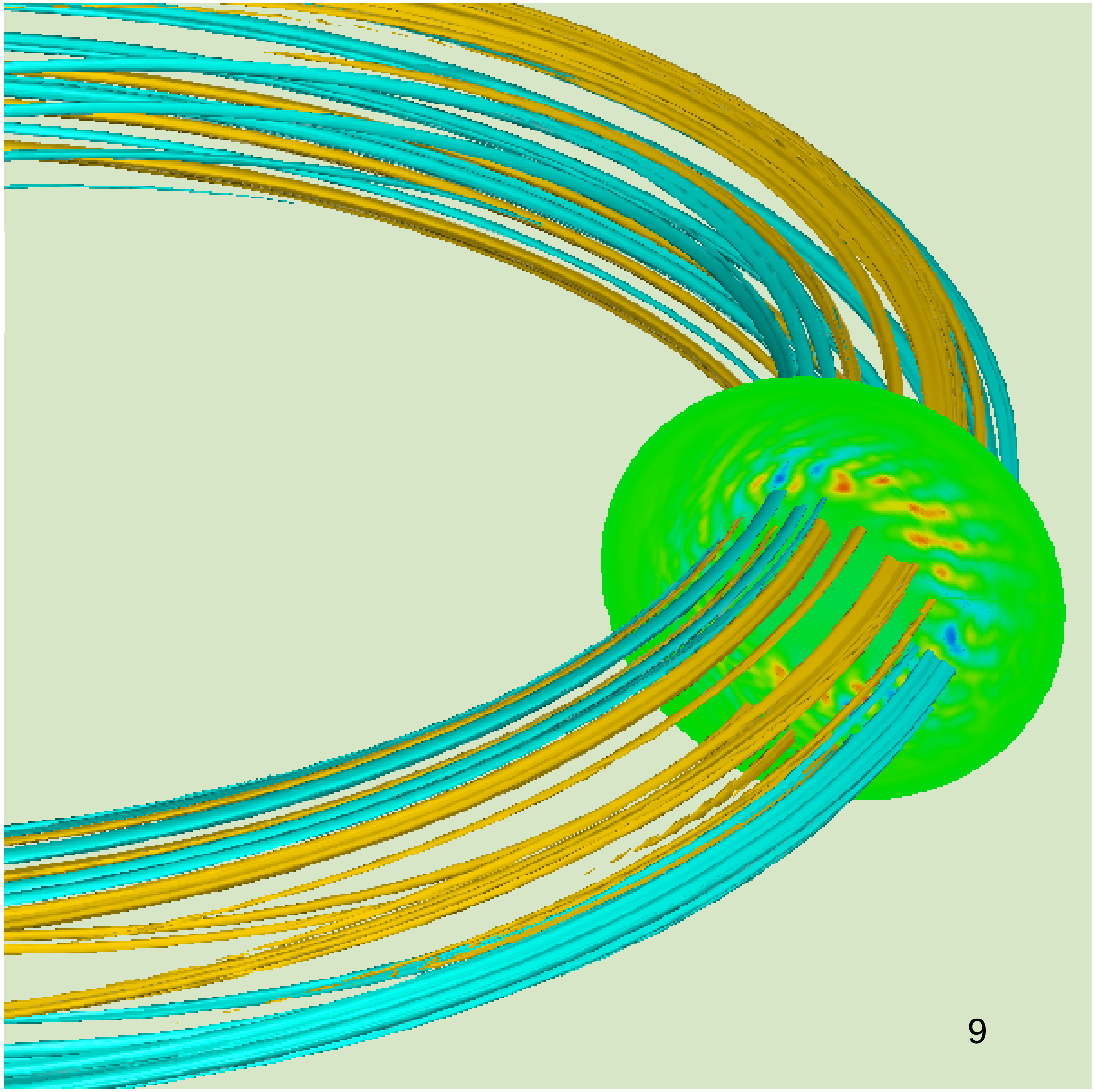
Spatial structure: Flute approximation

$$\mathbf{b} \cdot \nabla f / \nabla_\perp f \approx \varepsilon \ll 1$$

$$k_{//} / k_\perp \approx \varepsilon \ll 1$$

$$\nabla_\perp = \nabla - \mathbf{b} \cdot \nabla$$

$$\mathbf{b} = \mathbf{B}/\mathrm{B}$$



Time scale: Drift ordering

- MHD ordering $v_{\mathrm{E}} \simeq v_{\mathrm{Ti}}$
  – Perturbed electric field is so strong that ExB flow velocity is comparable with the ion thermal velocity.
- Drift ordering $v_{\mathrm{E}} \simeq \varepsilon v_{\mathrm{Ti}}$
  – Perturbed electric field is weak and ExB flow velocity is much smaller than the ion thermal velocity

Variables in GK equation

$$f_s = F_{Ms} + \delta f_s$$

$$F_{Ms} = \frac{n_0}{(2\pi T_s / m_s)^{3/2}} \exp\left(-\frac{m_s v_{//}^2}{2T_s} - \frac{\mu B}{T_s} \right)$$

$$\delta f_s(\mathbf{X}, v_{//}, \mu) = \sum_k \delta f_{sk}(k_x, k_y, z, v_{//}, \mu) \exp(iS_k)$$

$$\nabla S_k = \mathbf{k}_\perp = (k_x, k_y) \qquad \mathbf{v} = v_{//}\mathbf{b} + \mathbf{v}_\perp$$

EM delta-f gyrokinetic equations

$$\frac{D\delta f_{sk}}{Dt} + v_{Ts} v_{//} \mathbf{b}^* \cdot \nabla \delta f_{sk} - v_{Ts} \mu \mathbf{b} \cdot \nabla B \frac{\partial \delta f_{sk}}{\partial v_{//}} = -i\mathbf{v}_{ds} \cdot \mathbf{k}_\perp (\delta f_{sk} + \frac{q_s F_{sM}}{T_s} \phi_k J_{0s})$$

$$+ i\mathbf{v}_{*s} \cdot \mathbf{k}_\perp \frac{q_s F_{sM}}{T_s} (\phi_k - v_{Ts} v_{//} A_{//k}) J_{0s} + v_{Ts} v_{//} \frac{q_s F_{sM}}{T_s} E_{//k} + C(\delta f_{sk})$$

$$\lambda_{Di}^2 k_\perp^2 \phi_k = \sum_s \left(q_s \delta \hat{n}_{sk} - \frac{q_s^2}{T_s} [1 - \Gamma_{0s}] \phi_k \right)$$

$$k_\perp^2 A_{//k} = \beta_i \sum_s q_s \delta \hat{u}_{sk}$$

$$\delta \hat{n}_{sk} = \int dv^3 \delta f_{sk} J_{0s}$$

$$\delta \hat{u}_{sk} = \int dv^3 v_{//} \delta f_{sk} J_{0s}$$

$$E_{//k} = -\mathbf{b}^* \cdot \nabla \phi_k J_{0s} - \frac{\partial A_{//k}}{\partial t} J_{0s}$$

$$\mathbf{v}_{ds} = \frac{1}{q_s B} (\mu \nabla B + m_s v_{//}^2 \mathbf{b} \cdot \nabla \mathbf{b})$$

$$\mathbf{v}_{*s} = \frac{T_s}{q_s B} \mathbf{b} \cdot \nabla \ln F_{Ms}$$

Nonlinear terms

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{1}{B} [\phi J_{0s}, \quad]_k$$

$$\mathbf{b}^* \cdot \nabla = \mathbf{b} \cdot \nabla - \frac{1}{B} [A_{//} J_{0s}, \quad]_k$$

Difficulties in electromagnetic gyrokinetic simulations

- Cancellation problem
- Kinetic and adiabatic electrons
- Waves with high frequency

Cancellation problem

- Time derivative of A_// in E_// causes a problem

$$\frac{\partial}{\partial t}\left(\delta f_{sk} + v_{//}\frac{q_s F_{sM}}{cT_s} J_{0s} A_{//k}\right) = \frac{\partial \delta f_{sk}^{(h)}}{\partial t} = \cdots$$

$$(k_\perp^2 + \underbrace{\frac{4\pi}{c^2} n_0 \sum_s q_s^2 \Gamma_{0s}}_{\text{Very large}}) A_{//k} = \frac{4\pi}{c}\sum_s q_s \int dv^3 v_{//} \delta f_{sk}^{(h)} J_{0s}$$

- How to resolve the problem
  - Numerically integrate F_M in the Ampere's law

$$k_\perp^2 A_{//k} = \frac{4\pi}{c}\sum_s q_s \int dv^3 v_{//} \delta f_{sk}^{(h)} J_{0s} - \frac{4\pi}{c}\sum_s q_s \int dv^3 v_{//}^2 \frac{q_s F_{sM}}{cT_s} J_{0s}^2 A_{//k}$$

Kinetic and adiabatic electrons

- Difficulties in solving gyrokinetic equation for electrons because of small electron mass
  - Numerical oscillation occurs between passing and trapped regions.
  - Elongated mode structure along the field line
- Adiabatic electron approximation
  - Beta is set to be zero, so that magnetic perturbation vanishes.
  - V_te → infinity
  - Computational cost is significantly reduced, because we don't need to solve electron GK eq.

$$v_{//}\mathbf{b}\cdot\nabla\delta f_{ek} - \cancel{\mu\mathbf{b}\cdot\nabla B\frac{\partial\delta f_{ek}}{\partial v_{//}}} = -v_{//}\frac{q_e F_{eM}}{T_e}\nabla\phi_k \Rightarrow \delta n_{ek} = \frac{F_{eM}}{T_e}\phi_k \quad k_y \neq 0$$

Waves in homogeneous plasmas

Neglecting drift terms

$$\frac{\partial \delta f_{sk}}{\partial t} + v_{//}\mathbf{b}\cdot\nabla\delta f_{sk} = v_{//}\frac{q_s F_{sM}}{T_s} J_{0s} E_{//}$$

$$-k_\perp^2 \phi_k = 4\pi \sum_s q_s \delta n_{sk}$$

$$1-\Gamma_{0i} \approx \rho_{Ti}^2 k_\perp^2$$

$$k_\perp^2 A_{//k} = \frac{4\pi}{c}\sum_s q_s \delta u_{sk}$$

Dispersion relation

$$-\rho_i^2 k_\perp^2 = \left(\left(\frac{\omega}{v_A k_{//}}\right)^2 - 1\right)\left[\sum_s \frac{T_e}{T_s}(1+\zeta_s Z(\zeta_s))\right]$$

$$\zeta_s = \frac{\omega}{v_{Ts} k_{//}\sqrt{2}}$$

- Kinetic Alfven wave $k_{//}v_{Ti} << \omega << k_{//}v_{Te}$
  - k_perp term reduces magnetic field line bending stabilization, so that KBM is destablilized.

$$\omega^2 = (1-\rho_i^2 k_\perp^2)(v_A k_{//})^2$$

- High frequency mode $k_{//}v_{Te} << \omega << k_{//}v_A$ $(\beta_e << m_e/m_i)$
  - This mode restrict time step size of simulation in low beta.

$$\omega^2 = \left(\frac{k_{//}}{k_\perp}\Omega_i\right)^2 \frac{m_i}{m_e}$$

Instabilities

Parity

LINEAR ANALYSIS

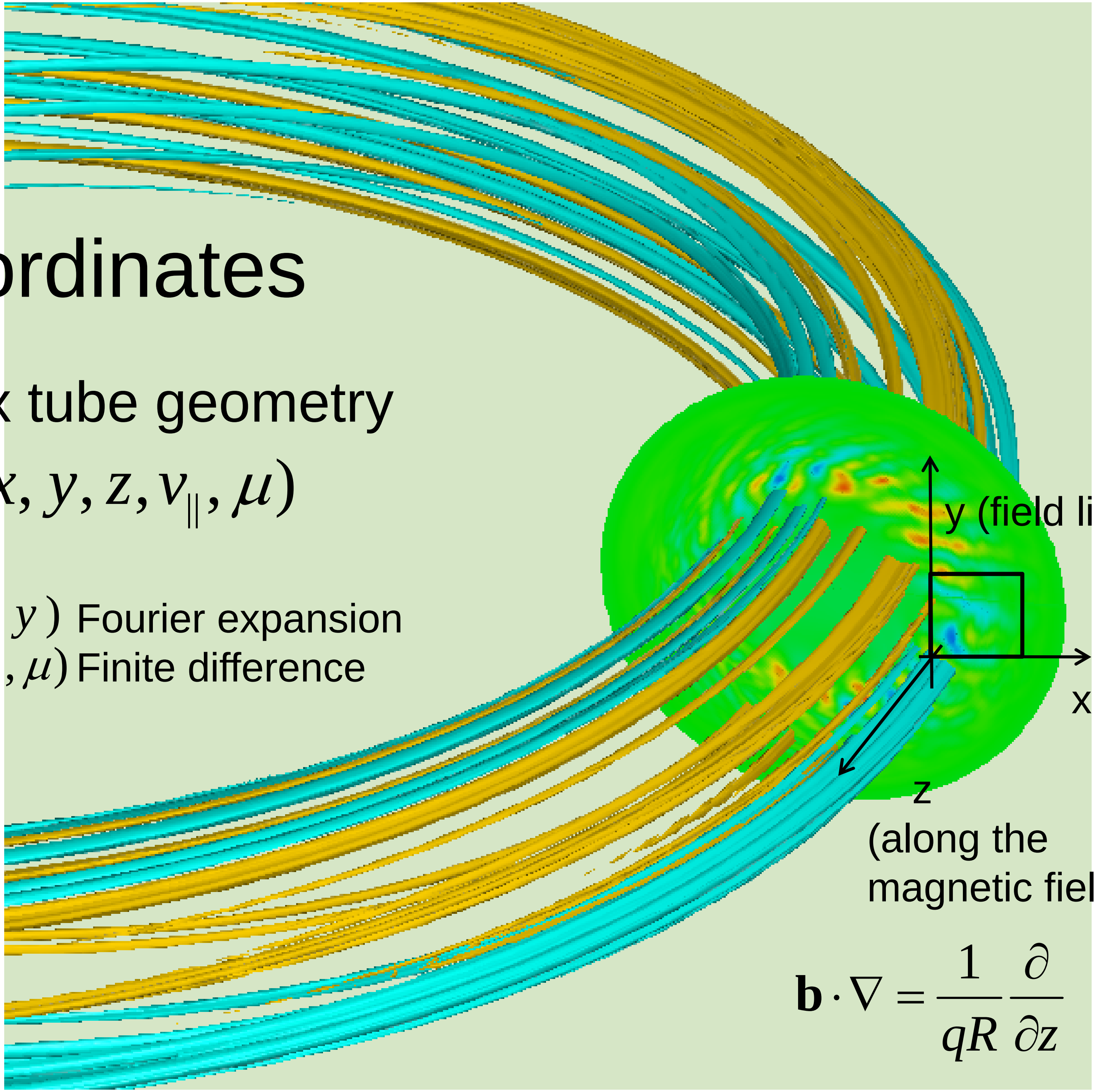
Coordinates

Flux tube geometry

$$(x, y, z, v_{\parallel}, \mu)$$

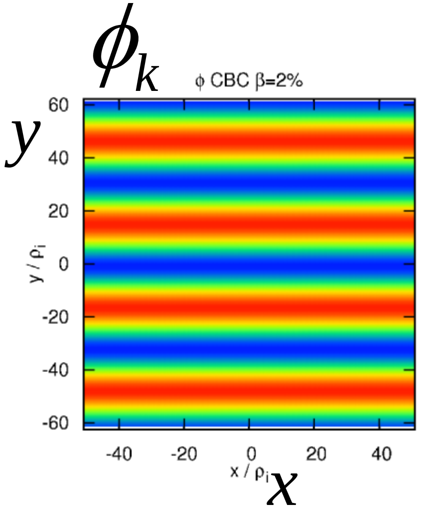
(x, y) Fourier expansion

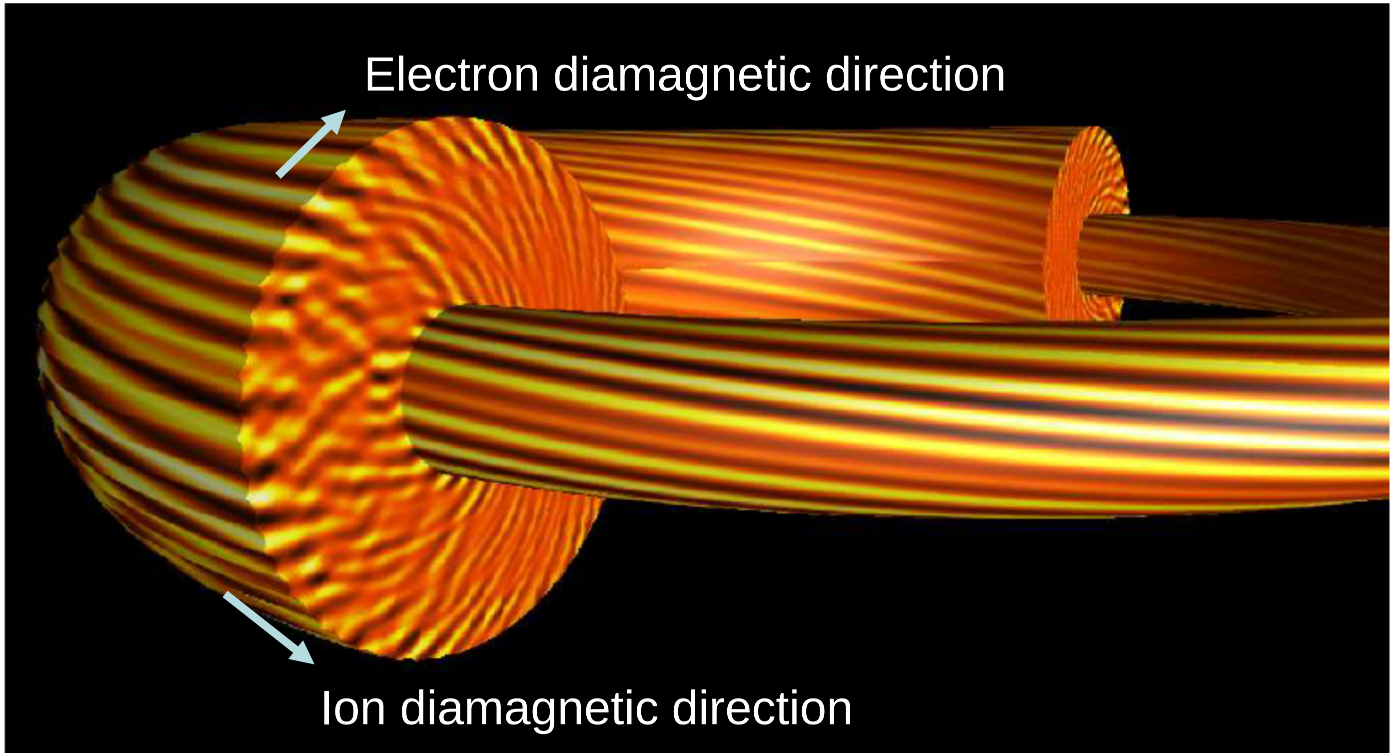
$(z, v_{\parallel}, \mu)$ Finite difference



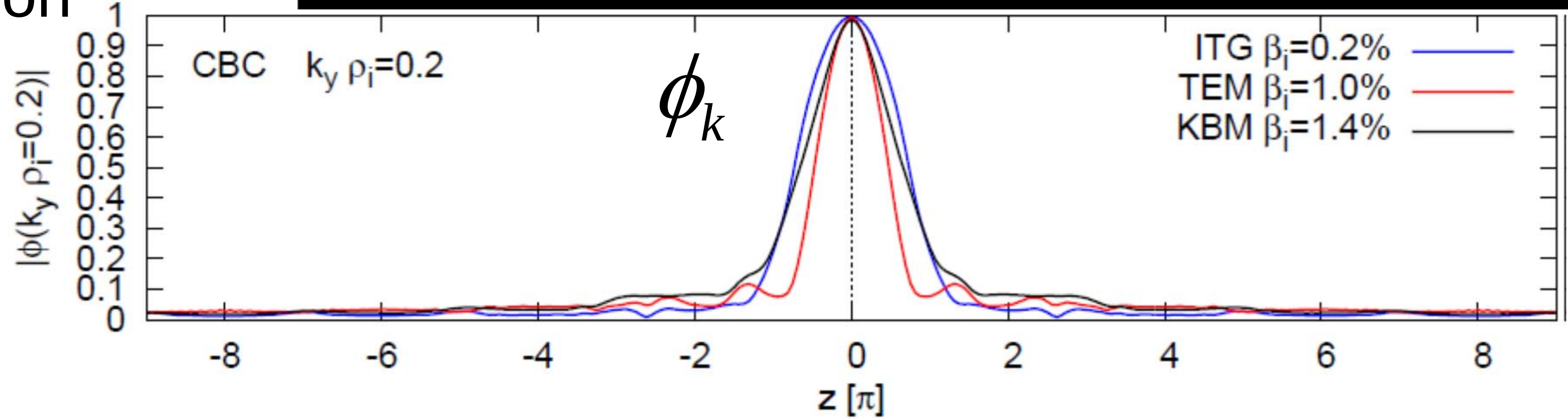
$$\mathbf{b} \cdot \nabla = \frac{1}{qR} \frac{\partial}{\partial z}$$

Ion temperature gradient (ITG) mode at $\beta_i = 0.2\%$





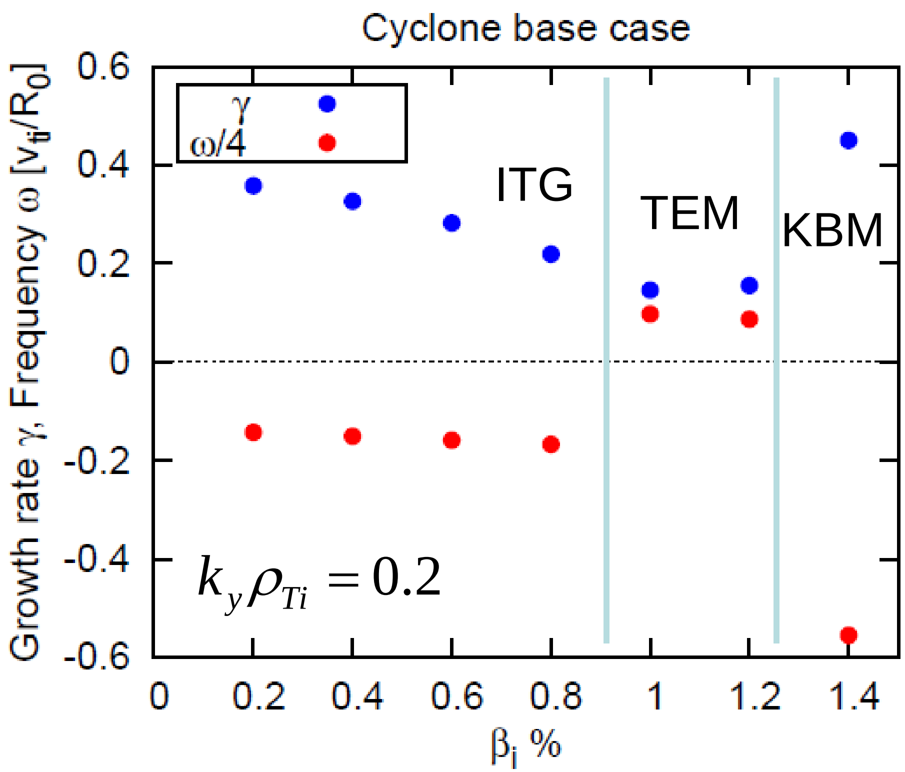
ITG rotates in the ion diamagnetic direction

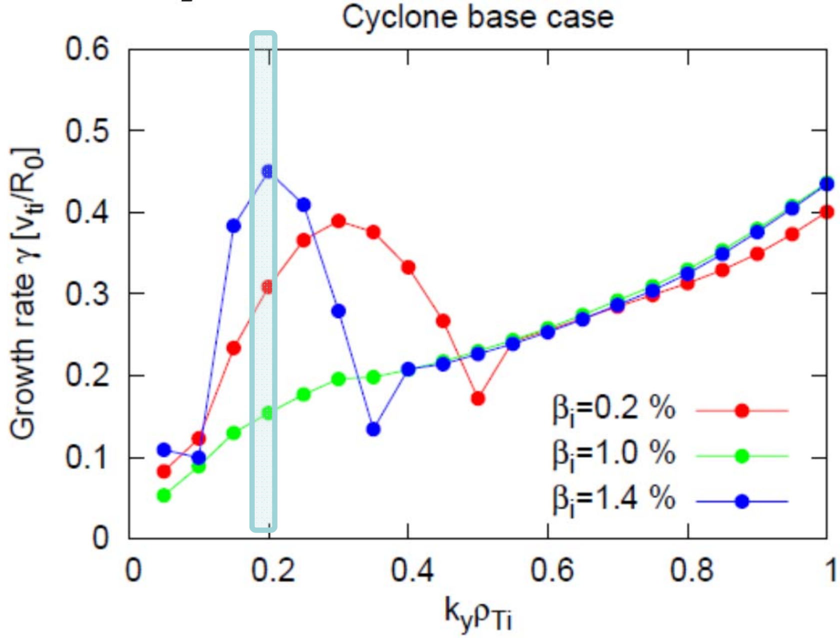


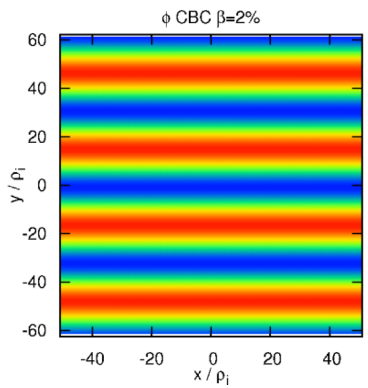
Instabilities which drive micro-turbulence

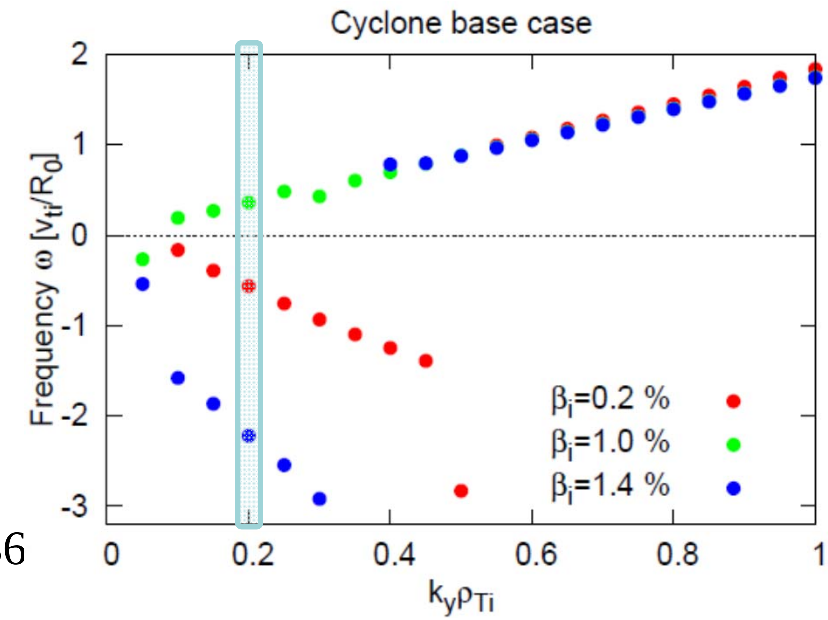
- Drift wave instability
  - Ion temperature gradient (ITG) mode
  - Trapped electron mode (TEM)
- Electromagnetic instability
  - Kinetic ballooning mode (KBM)
  - Micro-tearing mode (MTM)

Linear analysis









$q_0 = 1.4$

$\hat{s} = 0.786$

Parity

- The linearized equation is invariant for

$$z \to -z,\ v_{//} \to -v_{//},\ \theta_k \to -\theta_k \qquad \theta_k = -k_x/(k_y \hat{s})$$

$$\frac{\partial \delta f_{sk}}{\partial t} + v_{Ts} v_{//} \mathbf{b} \cdot \nabla \delta f_{sk} - v_{Ts} \mu \mathbf{b} \cdot \nabla B \frac{\partial \delta f_{sk}}{\partial v_{//}} = -i\mathbf{v}_{ds} \cdot \mathbf{k}_\perp (\delta f_{sk} + \frac{q_s F_{sM}}{T_s} \phi_k J_{0s})$$

$$+ i\mathbf{v}_{*s} \cdot \mathbf{k}_\perp \frac{q_s F_{sM}}{T_s} (\phi_k - v_{Ts} v_{//} A_{//k}) J_{0s} + v_{Ts} v_{//} \frac{q_s F_{sM}}{T_s} E_{//k} + C(\delta f_{sk})$$

- Ballooning parity

$$\delta f_{sk}(-z, -v_{//}, -\theta_k) = \delta f_{sk}(z, v_{//}, \theta_k)$$

$$\phi_k(-z, -\theta_k) = \phi_k(-z, -\theta_k)$$

$$A_{//k}(-z, -\theta_k) = -A_{//k}(-z, -\theta_k)$$

- Tearing parity

$$\delta f_{sk}(-z, -v_{//}, -\theta_k) = -\delta f_{sk}(z, v_{//}, \theta_k)$$

$$\phi_k(-z, -\theta_k) = -\phi_k(-z, -\theta_k)$$

$$A_{//k}(-z, -\theta_k) = A_{//k}(-z, -\theta_k)$$

Parity

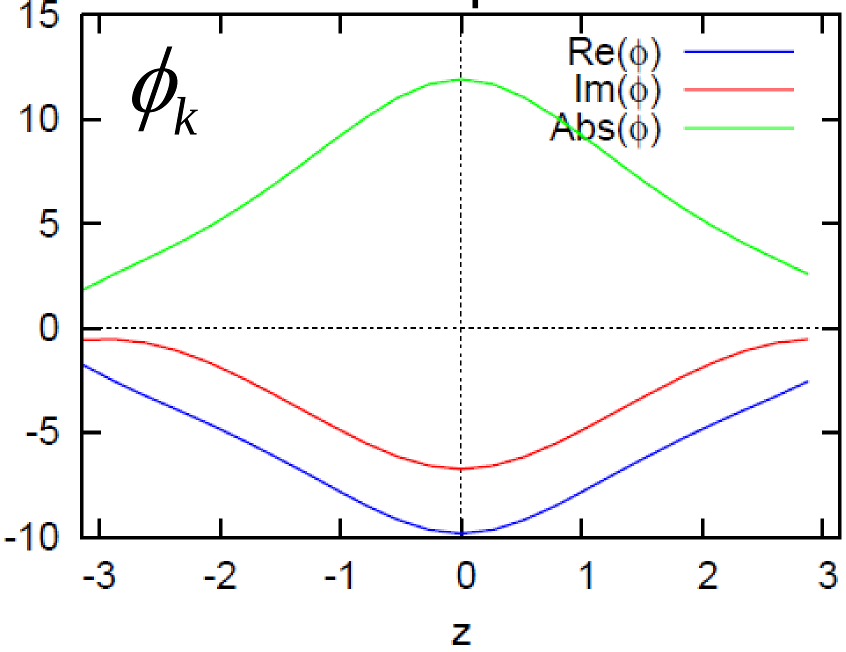
- Ballooning parity

$$\delta f_{sk}(-z,-v_{//},-\theta_k)=\delta f_{sk}(z,v_{//},\theta_k)$$

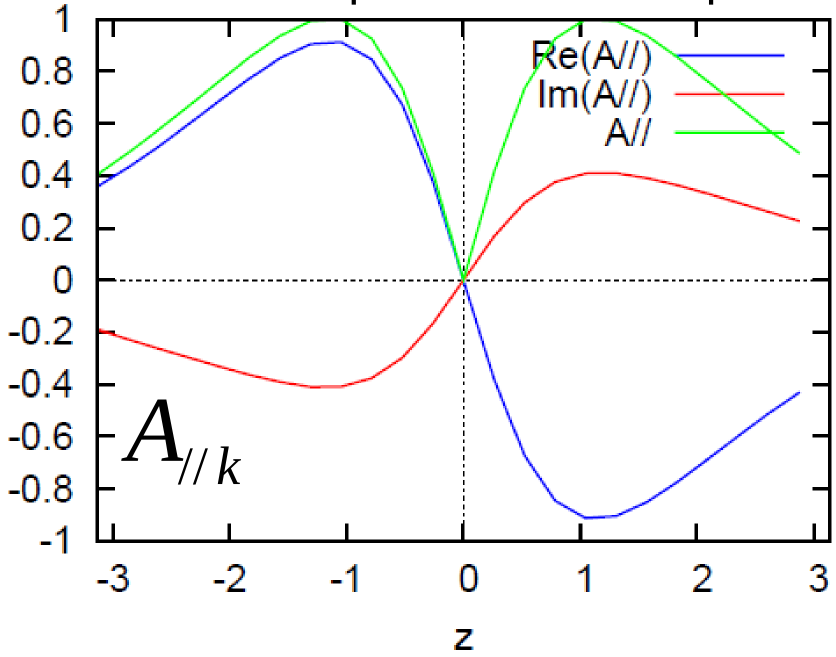
$$\phi_k(-z,-\theta_k)=\phi_k(-z,-\theta_k)$$

$$A_{//k}(-z,-\theta_k)=-A_{//k}(-z,-\theta_k)$$

Electrostatic potential

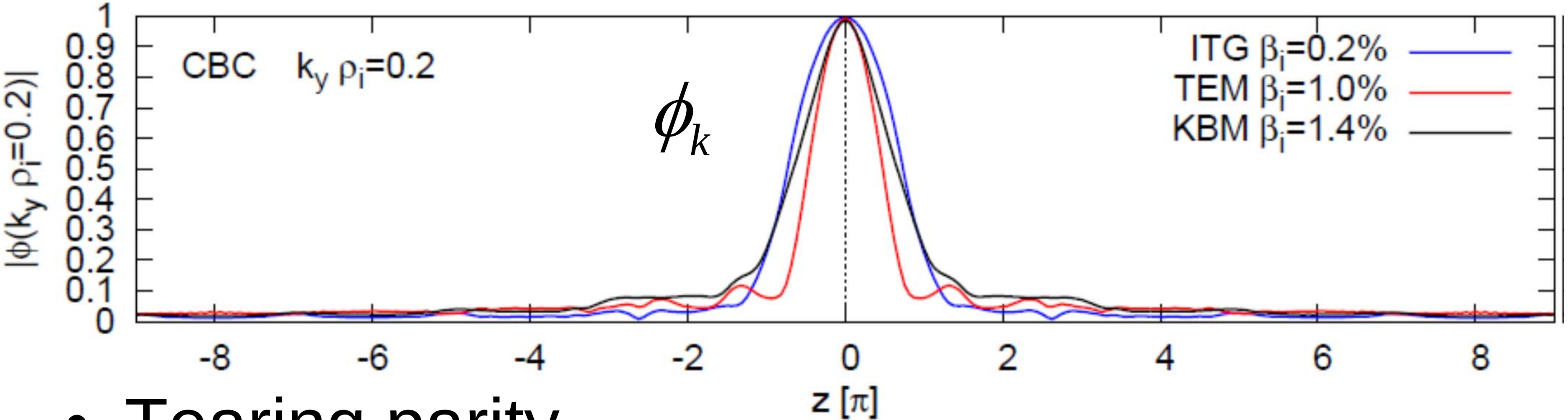


Parallel component of vector potential

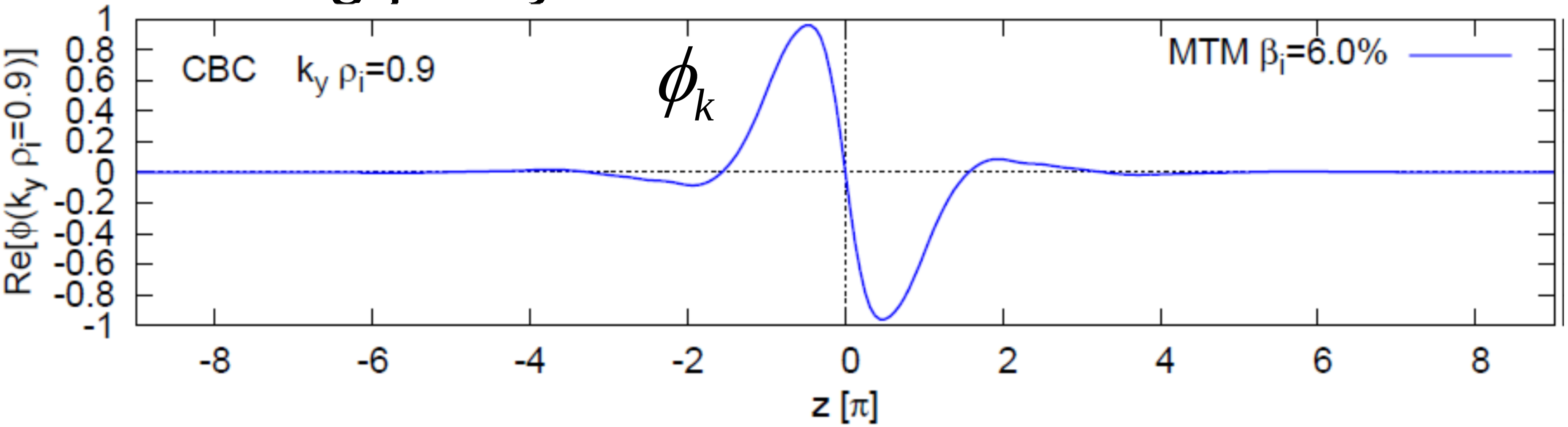


Parity

- Ballooning parity

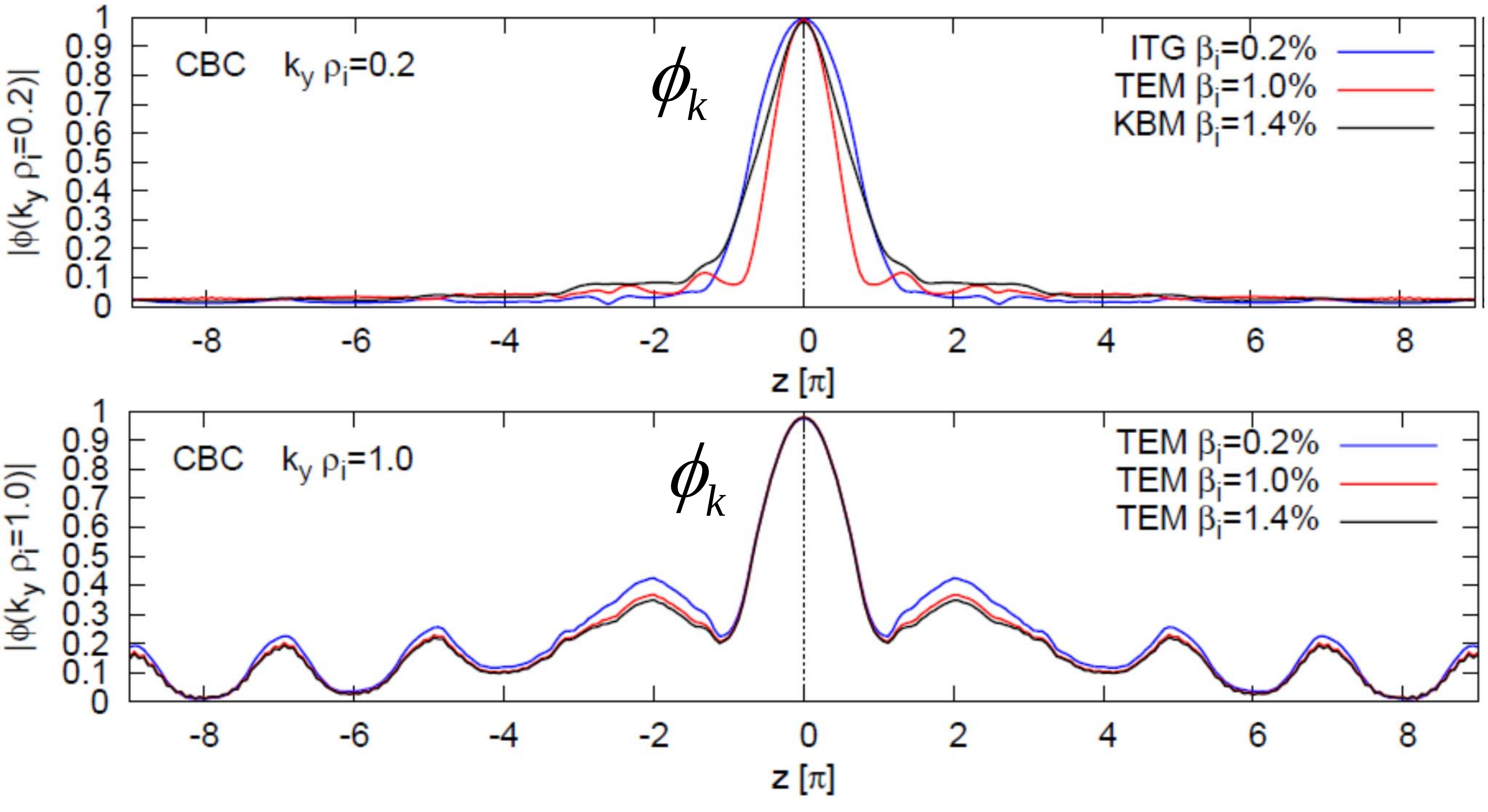


- Tearing parity



Trapped electron modes

- TEM can have elongated mode structure along the magnetic field line z.



Parity symmetry

Conservation of quadratic quantity (Entropy balance equation)

ANALYSIS OF NONLINEAR SIMULATION RESULTS

Parity exchange

- Nonlinear mixture of parities come from the Poisson bracket nonlinear term.

Ballooning parity $\delta f_{sk}(-z,-v_{//},-\theta_k)=\delta f_{sk}(z,v_{//},\theta_k)$

Tearing parity $\delta f_{sk}(-z,-v_{//},-\theta_k)=-\delta f_{sk}(z,v_{//},\theta_k)$

$$\frac{\partial \delta f_{sk}}{\partial t}+\boxed{\frac{1}{B}[(\phi-v_{//}A_{//})J_{0s},\delta f_s+\frac{q_sF_{sM}}{T_s}\phi]_k}+v_{Ts}v_{//}\mathbf{b}\cdot\nabla\delta f_{sk}$$

$$=-i\mathbf{v}_{ds}\cdot\mathbf{k}_\perp(\delta f_{sk}+\frac{q_sF_{sM}}{T_s}\phi_kJ_{0s})+v_{Ts}\mu\mathbf{b}\cdot\nabla B\frac{\partial\delta f_{sk}}{\partial v_{//}}$$

$$+i\mathbf{v}_{*s}\cdot\mathbf{k}_\perp\frac{q_sF_{sM}}{T_s}(\phi_k-v_{Ts}v_{//}A_{//k})J_{0s}+v_{Ts}v_{//}\frac{q_sF_{sM}}{T_s}E_{//k}+C(\delta f_{sk})$$

Entropy balance equation

$$\frac{d}{dt}\left(\sum_s \delta S_s + \delta W_{es} + \delta W_{em}\right) = \sum_s \left(\frac{\Theta_s}{L_{Ts}} + \frac{T_s \Gamma_s}{L_{ps}} + D_s\right)$$

$$\delta S_s = \left\langle \sum_k \int \frac{T_s |\delta f_{sk}|^2}{2F_{Ms}} d^3 v \right\rangle \qquad \delta W_{em} = \left\langle \sum_s \frac{k_\perp^2 |\delta A_{//k}|^2}{2\beta_i} \right\rangle$$

Heat flux $\Theta_s = \Theta_{es,s} + \Theta_{em,s}$

Particle flux $\Gamma_s = \Gamma_{es,s} + \Gamma_{em,s}$

$$\delta W_{es} = \left\langle \sum_k \left(\lambda_{Di}^2 k_\perp^2 + \sum_s \frac{q_s^2}{T_s}(1-\Gamma_{0s}) \right) \frac{|\delta\phi_k|^2}{2} \right\rangle$$

$$\Theta_{es,s} = \mathrm{Re}\left\langle \sum_k \left(\frac{\delta p_{//s}}{2} + \delta p_{\perp s} - \frac{5}{2} T_s \delta n_s \right) \frac{i k_y \delta\phi_k^*}{B} \right\rangle \qquad \Theta_{em,s} = \mathrm{Re}\left\langle \sum_k \left(\frac{\delta q_{//s}}{2} + \delta q_{\perp s} \right) \frac{-i k_y \delta A_{//k}^*}{B} \right\rangle$$

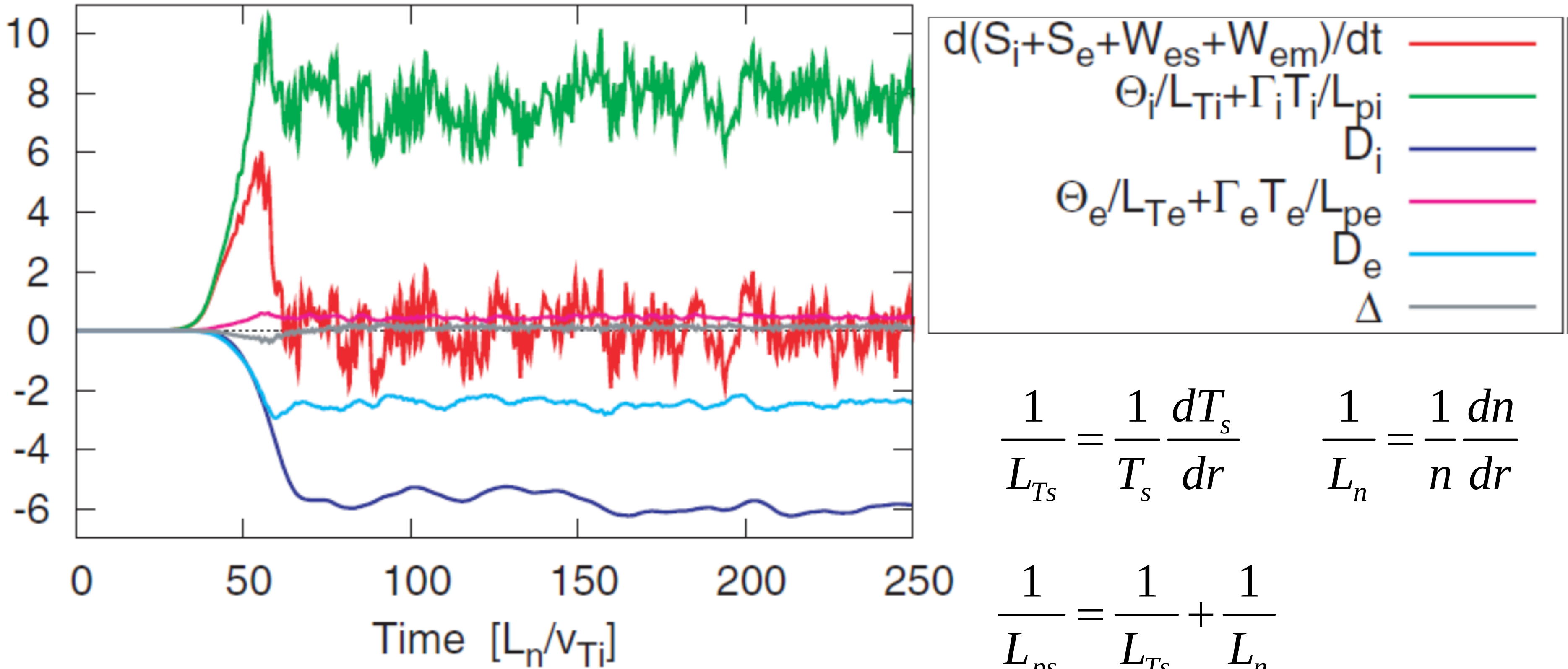
$$\Gamma_{es,s} = \mathrm{Re}\left\langle \sum_k \delta n_s \frac{i k_y \delta\phi_k^*}{B} \right\rangle \qquad \Gamma_{em,s} = \mathrm{Re}\left\langle \sum_k \delta u_s \frac{i k_y \delta A_{//k}^*}{B} \right\rangle$$

$$D_s = \nu_{ss} \left\langle \sum_k \int \left(\delta f_{sk} + \frac{q_s F_{sM}}{T_s} \phi_k J_{0s}\right)^* C(\delta f_{sk}) d^3 v \right\rangle$$

Sugama, Phys. Plasmas 2009

Entropy balance equation

$$\frac{d}{dt}\left(\sum_s \delta S_s + \delta W_{es} + \delta W_{em}\right) = \sum_s \left(\frac{\Theta_s}{L_{Ts}} + \frac{T_s \Gamma_s}{L_{ps}} + D_s\right)$$



$$\frac{1}{L_{Ts}} = \frac{1}{T_s}\frac{dT_s}{dr} \qquad \frac{1}{L_n} = \frac{1}{n}\frac{dn}{dr}$$

$$\frac{1}{L_{ps}} = \frac{1}{L_{Ts}} + \frac{1}{L_n}$$

Entropy transfer

- We can study the saturation mechanism of turbulence based on the conservation of quadratic quantities (entropy balance).

$$\frac{d}{dt}\left(\sum_s \delta S_{s,k} + W_{es,k} + W_{em,k}\right) = \sum_s \left(T_{s,k} + \frac{\Theta_{s,k}}{L_{Ts}} + \frac{T_s \Gamma_{s,k}}{L_{ps}} + D_{s,k}\right)$$

$$T_{s,k} = \sum_{k',k''} T_s(\mathrm{k};\mathrm{k}',\mathrm{k}'')$$

Entropy transfer function

$$T(\mathbf{k};\mathbf{k}',\mathbf{k}'') = \left\langle \int dv^3 \frac{T_s h_{sk}^*}{2F_{Ms}} \delta_{k,k'+k''} \mathbf{b}\cdot\mathbf{k}'\times\mathbf{k}''(\chi_{sk'} h_{sk''} - h_{sk'}\chi_{sk''}) \right\rangle$$

$$h_{sk} = f_{sk} + \frac{q_s}{T_s}\phi_k J_{0s} F_{Ms} \qquad \chi_{sk} = (\phi_k - v_{Ts} v_{//} A_{//k}) J_{0s}$$

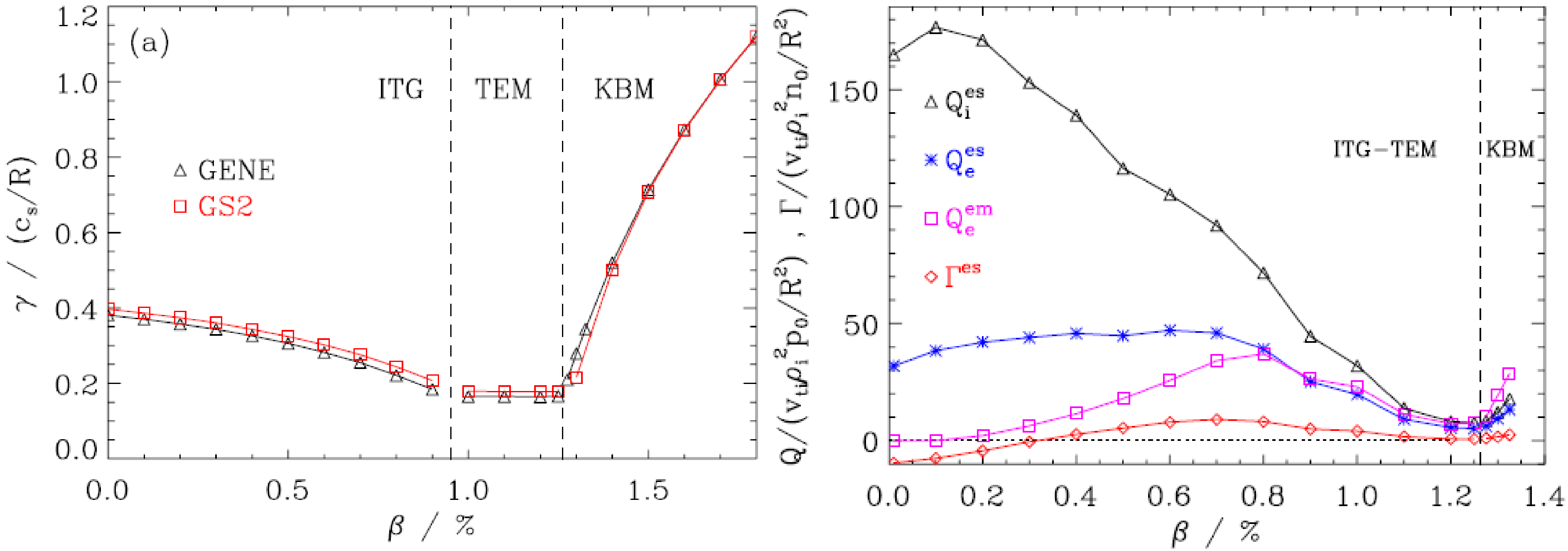
$$T(\mathrm{k};\mathrm{k}',\mathrm{k}'') + T(\mathrm{k}';\mathrm{k}'',\mathrm{k}) + T(\mathrm{k}'';\mathrm{k},\mathrm{k}') = 0$$

Beta dependence of turbulent transport

Saturation of turbulence in finite-beta plasmas

NONLINEAR SIMULATIONS

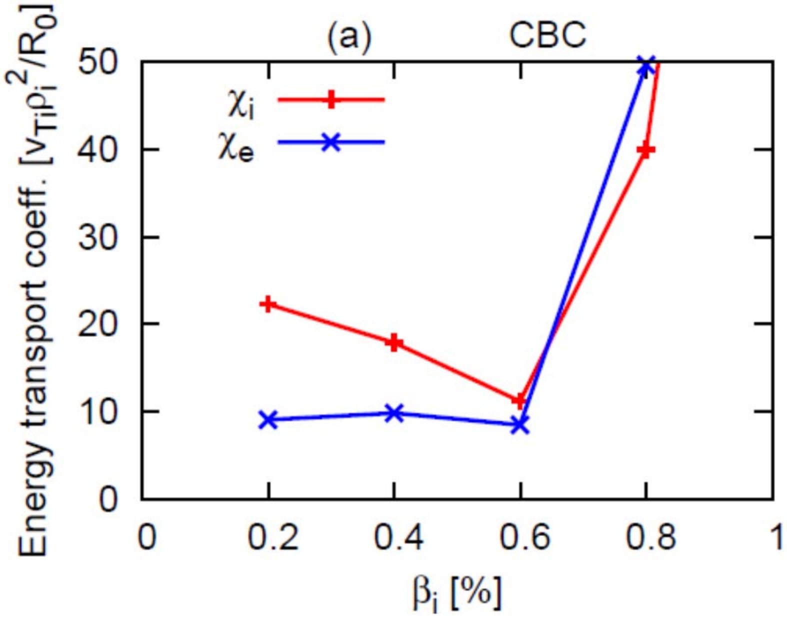
Beta dependence of turbulent transport

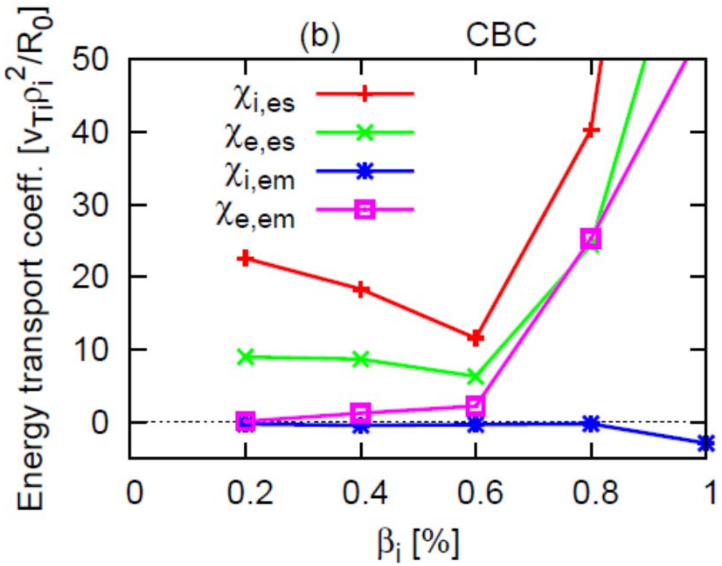


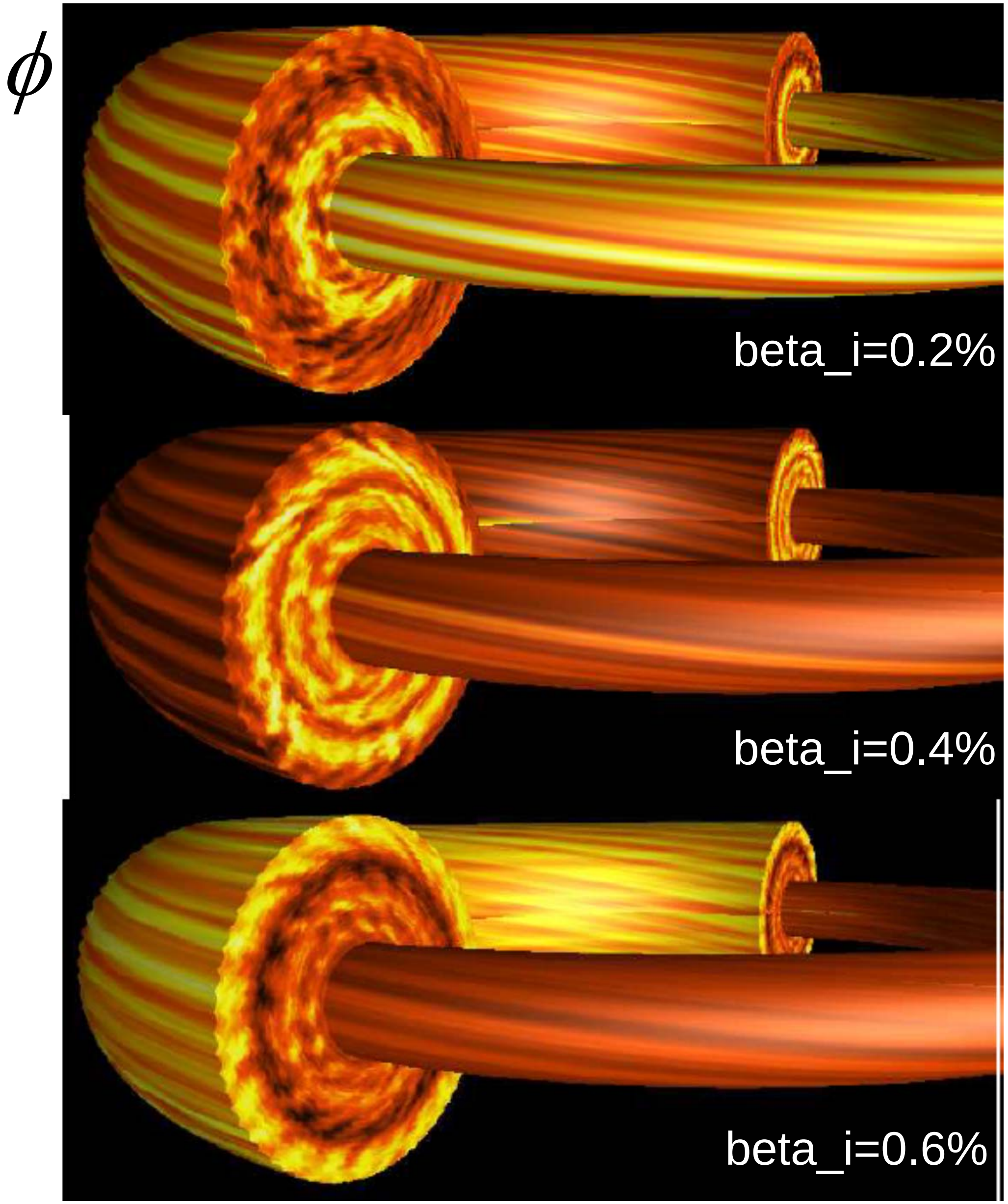
- Transport decreases faster than the linear growth rate.

M. J. Pueschel, PoP (2008)

Beta dependence of turbulence

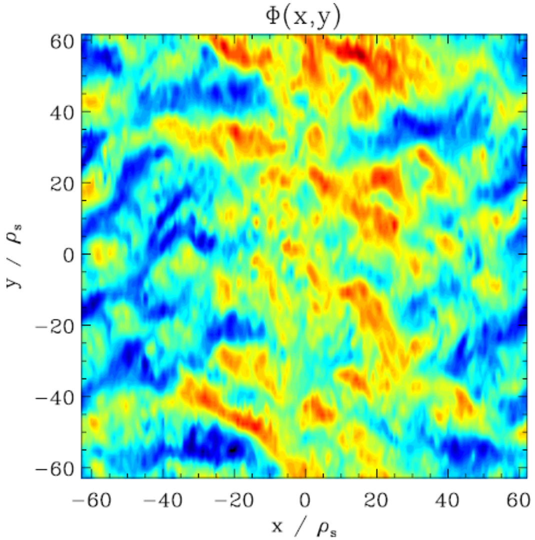




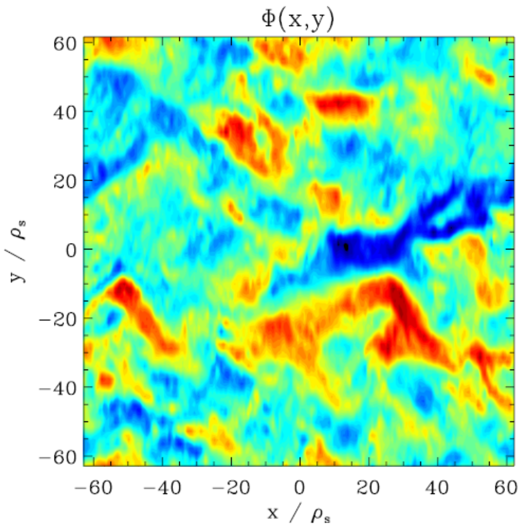


Dimits shift in EM

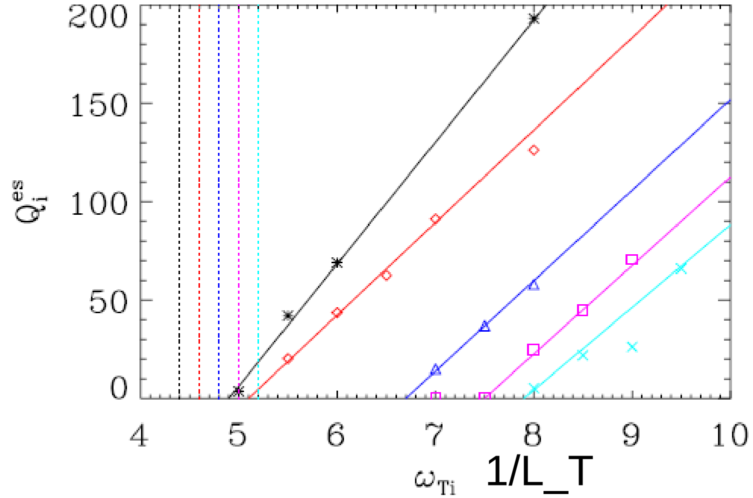
- Electromagnetic TEM/ITG turbulence



beta=0.01% (low beta)



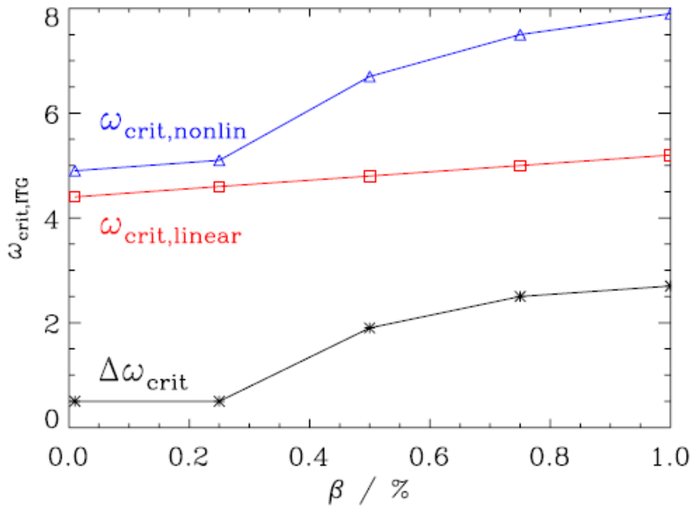
beta=1.75% (finite beta)



$$Q \propto \left(\frac{1}{L_T} - \frac{1}{L_{T,crit}} \right)$$

$$\frac{1}{L_T} = \frac{1}{T}\frac{dT}{dr}$$

The shift becomes large at higher beta

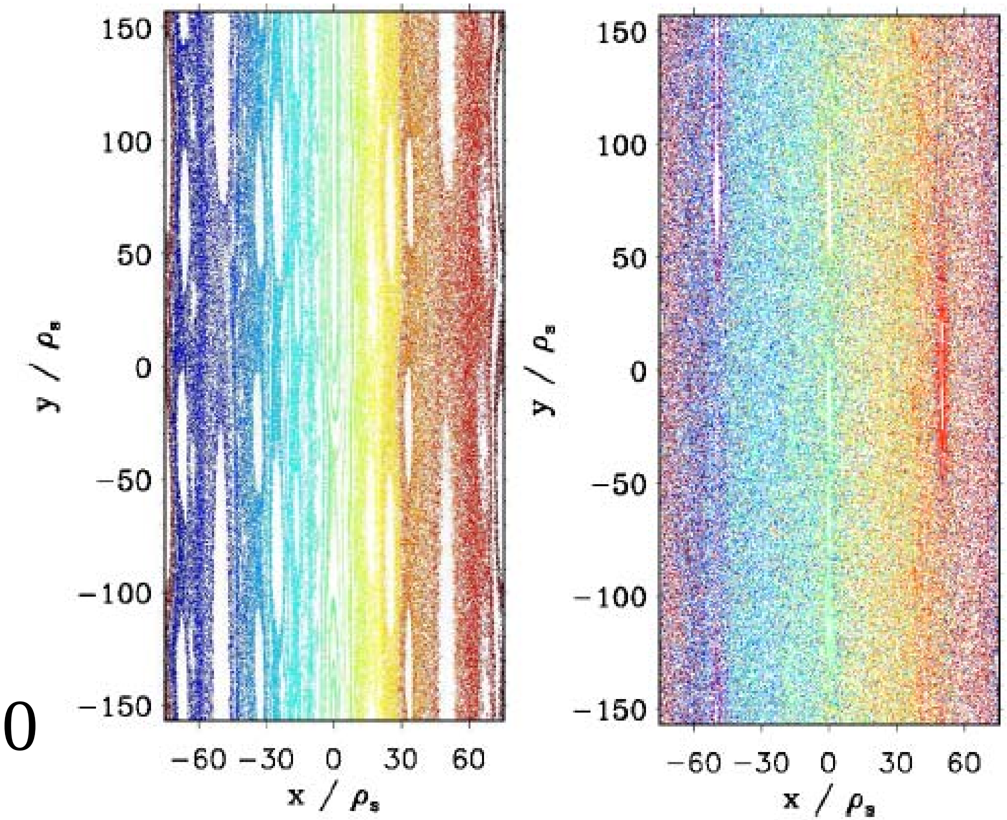


M. J. Pueschel, PoP (2010)

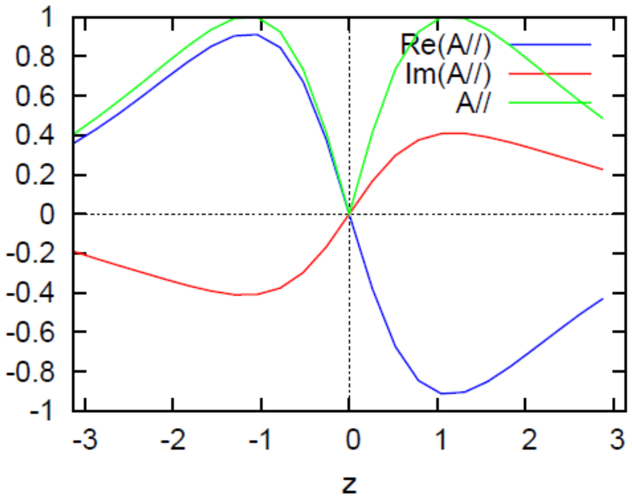
Transport by magnetic perturbation

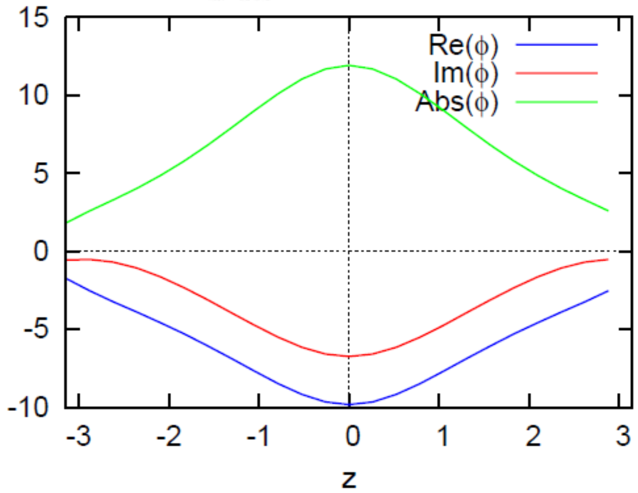
- Parity exchange causes stochastic magnetic field

$$\int \tilde{B}_x dz = \int i k_y A_{//k} dz \neq 0$$



$A_{//k}$



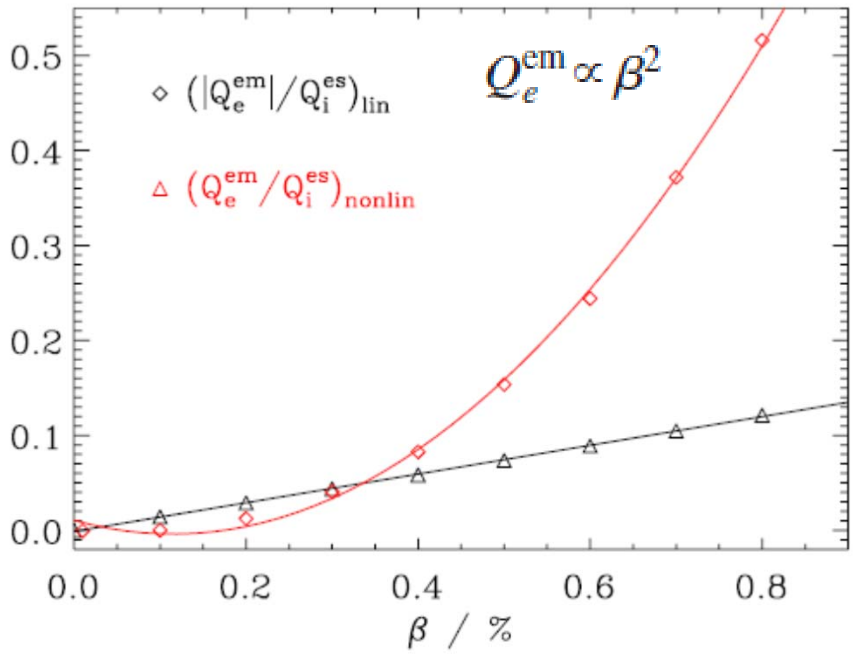


H. Doerk, et.al, PRL (2011).

Transport by magnetic perturbation

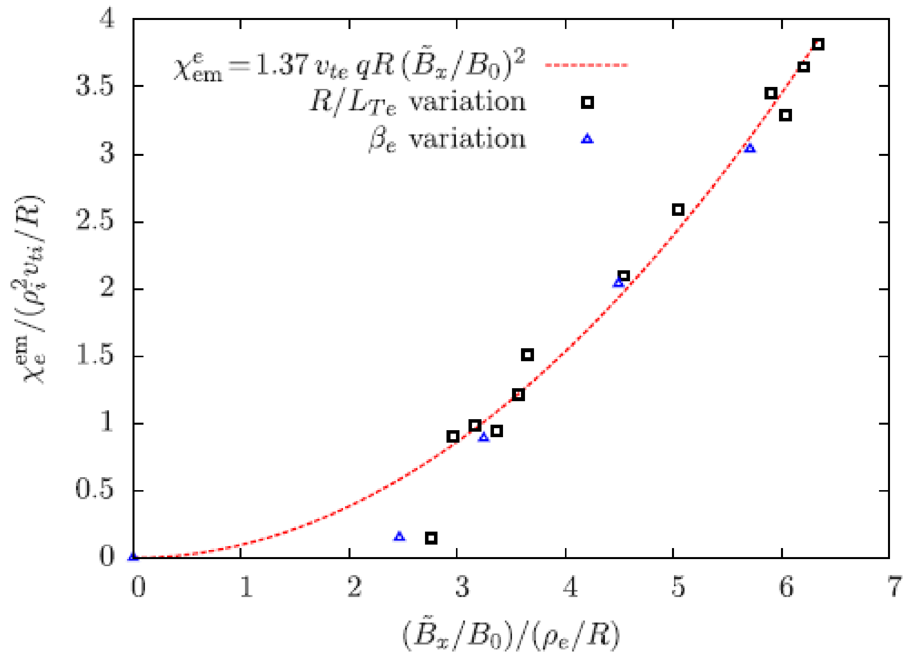
- ITG produces stable tearing parity modes and causes stochastic magnetic field.
- Magnetic flutter

$$\mathbf{b}^{*}\cdot\nabla=\mathbf{b}\cdot\nabla-\frac{1}{B}[A_{//}J_{0s},\quad]_{k}$$



Pueschel, Phys. Plasmas, (2008)
D. Hatch, PRL, (2012)

- Micro-tearing mode (MTM)
- The Rechester-Rosenbluth model is in good agreement at high amplitude, while the model breaks down at small amplitude.



H. Doerk, et.al, Phys. Rev. Lett. (2011).

Zonal flows are weak in finite-beta plasmas

SATURATION PROBLEM IN EMGK

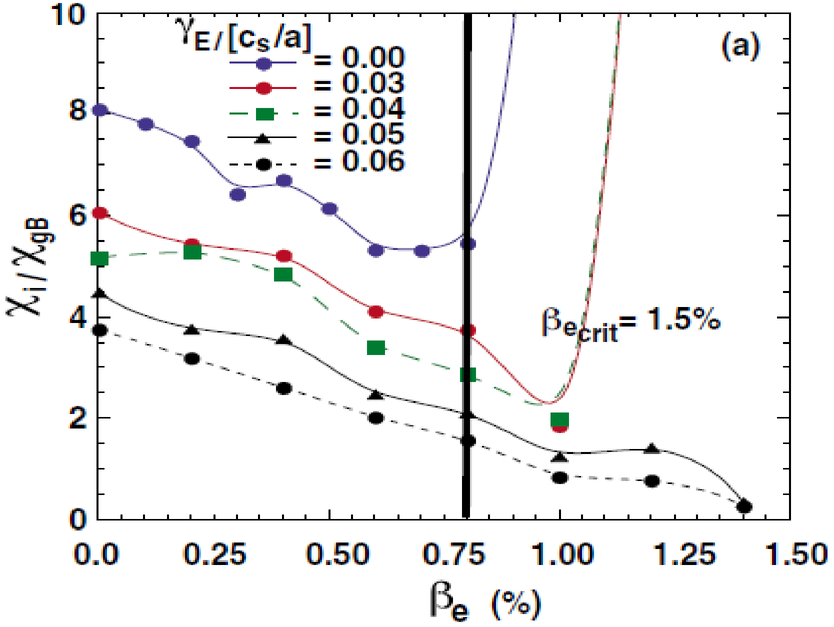
Saturation problem in finite beta plasmas

Failure of the transport levels to saturate at finite beta in gyrokinetic simulations in flux tube geometry

Cyclone base case (tokamak)

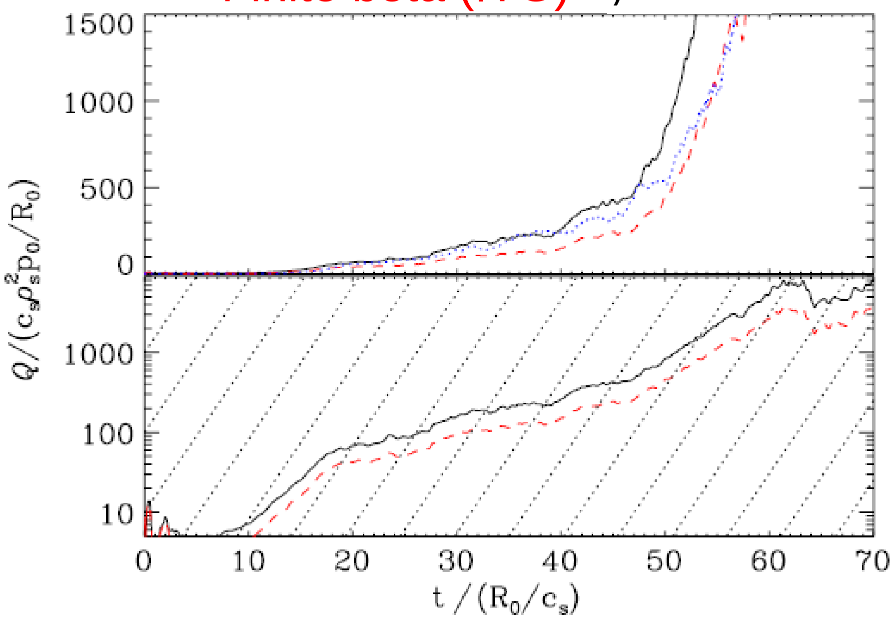
Finite beta (ITG)

Runaway above $\beta_e = 0.75\%$



R. E. Waltz, Phys. Plasmas, (2010)

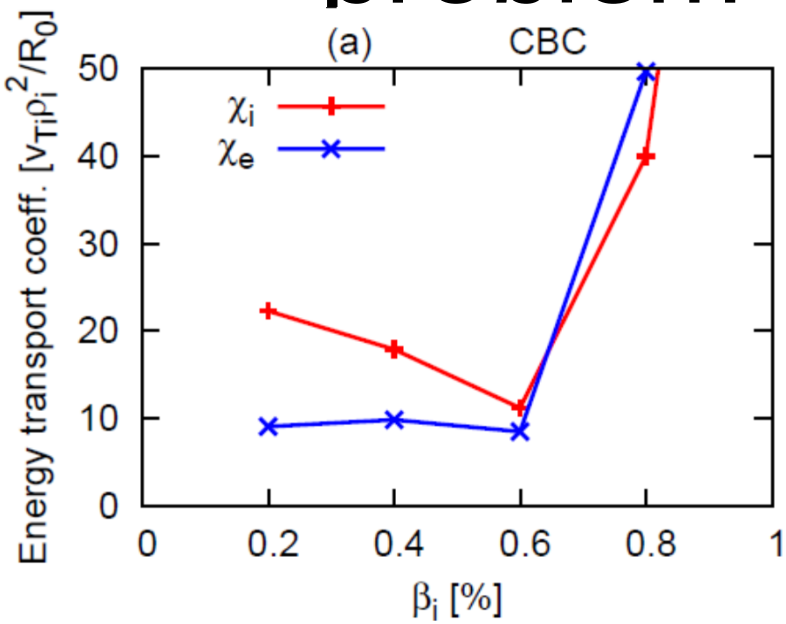
Finite beta (ITG) $\beta = 0.9\%$

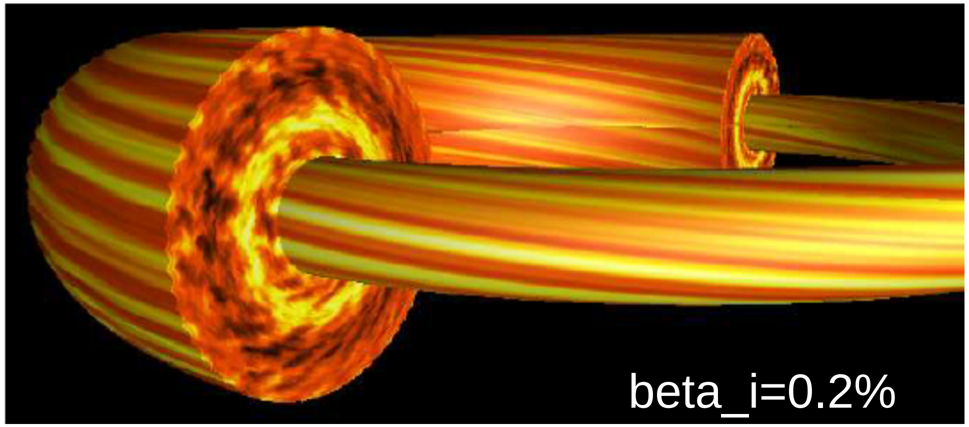


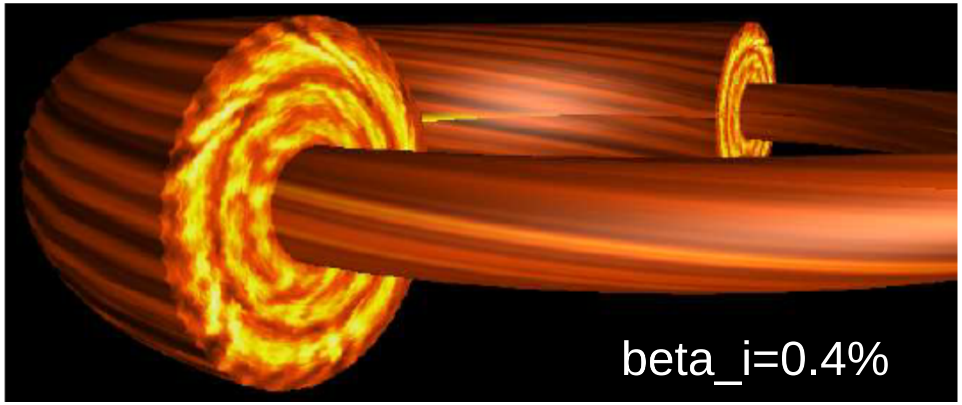
M. J. Pueschel, Phys. Rev. Lett., (2013)

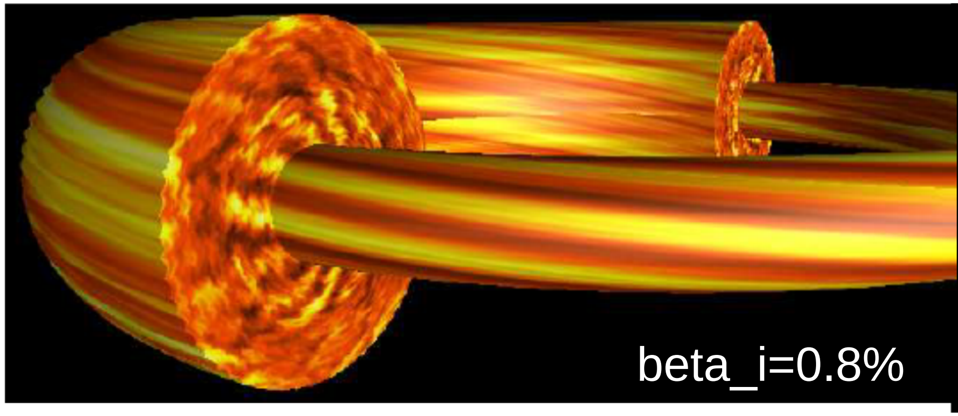
Zonal flows are weak.

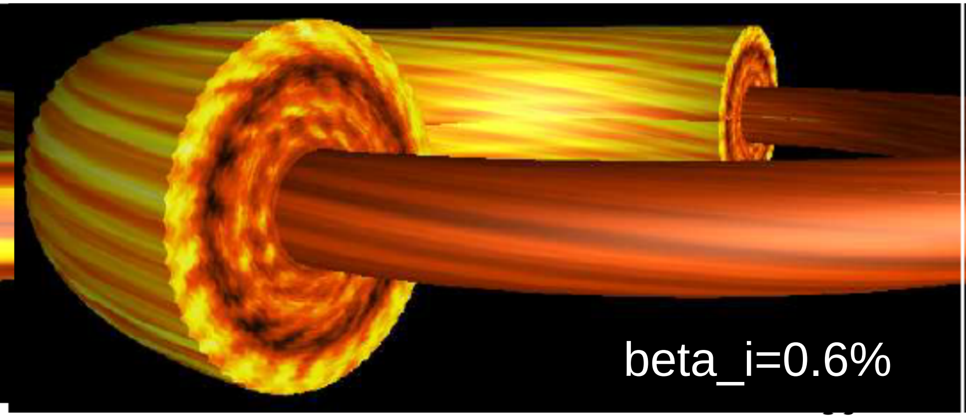
Saturation problem



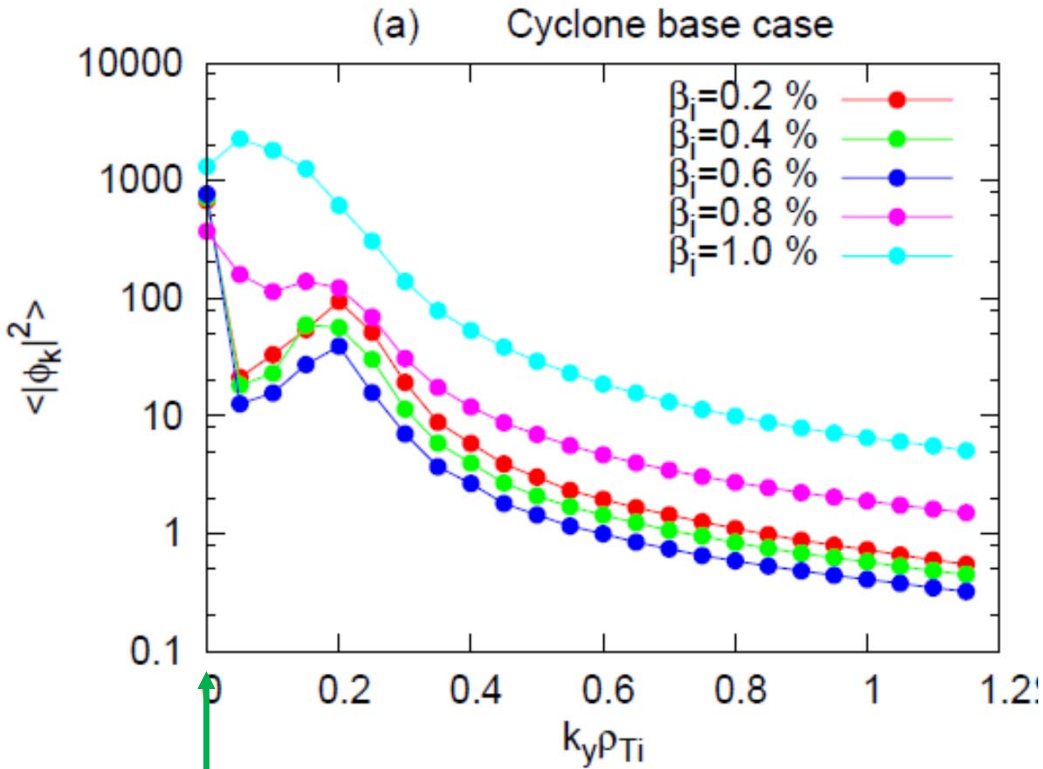




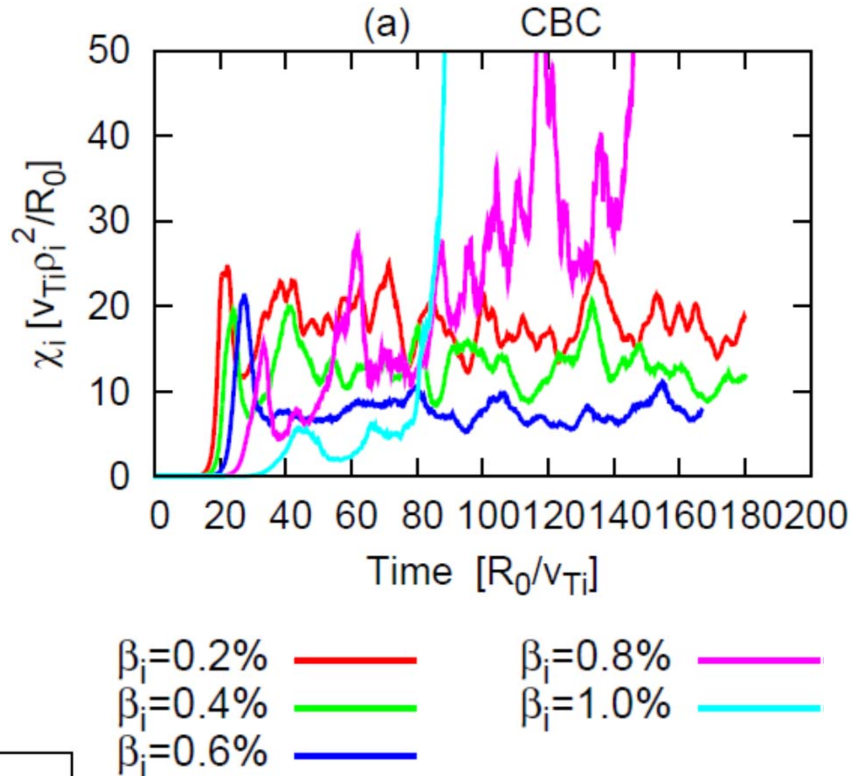


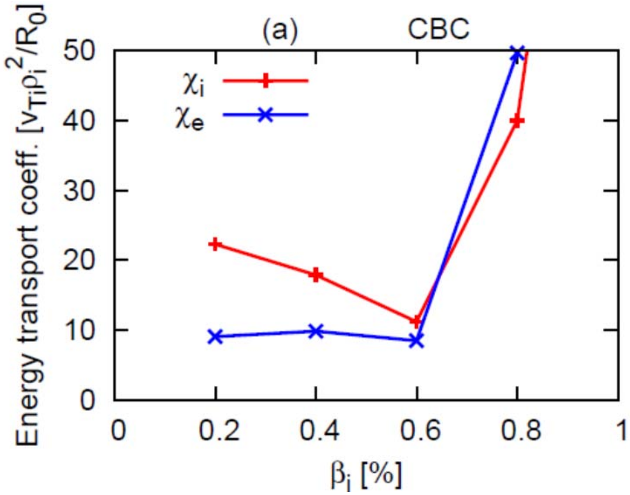


Saturation problem



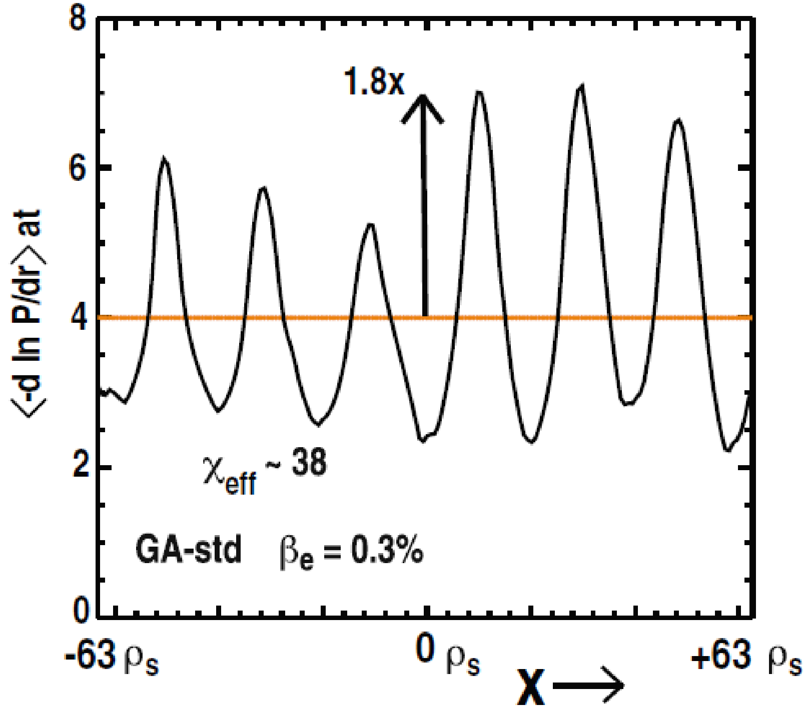
Zonal flow





Possible mechanisms of the transition

- Nonlinear subcritical instability due to the increase of local pressure gradient.



Waltz, Phys. Plasmas (2010)

A saturated state of turbulence at high beta by adopting small electron temperature gradient

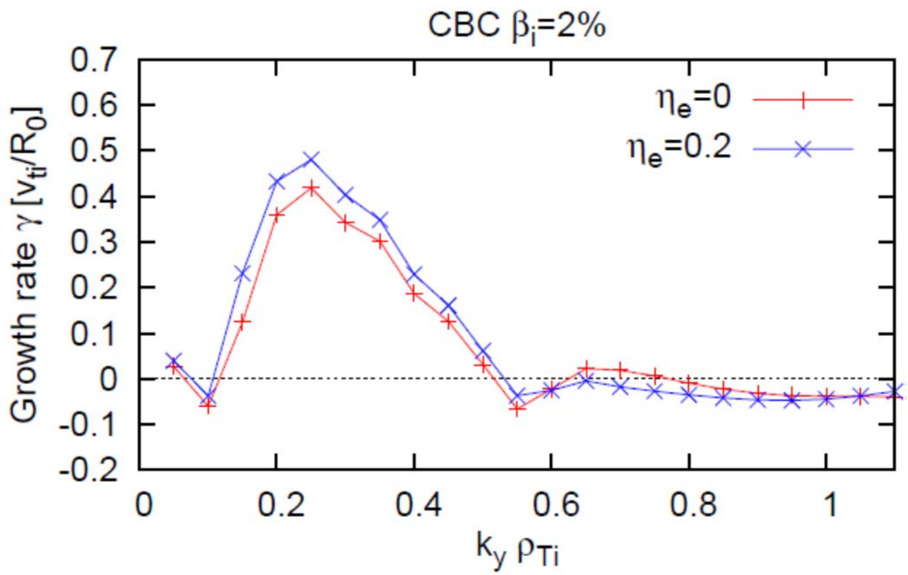
Entropy transfer analysis

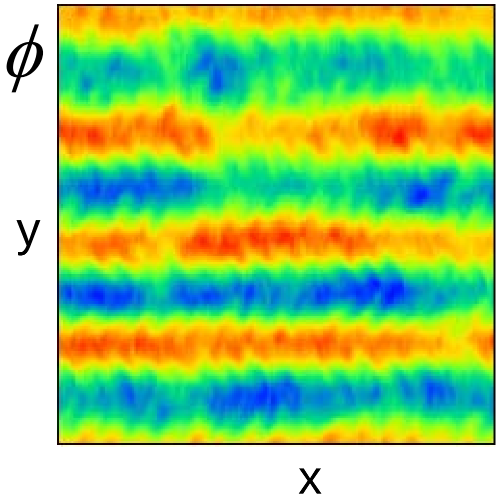
TURBULENCE AT HIGH-BETA WITH SMALL ELECTRON TEMPERATURE GRADIENT

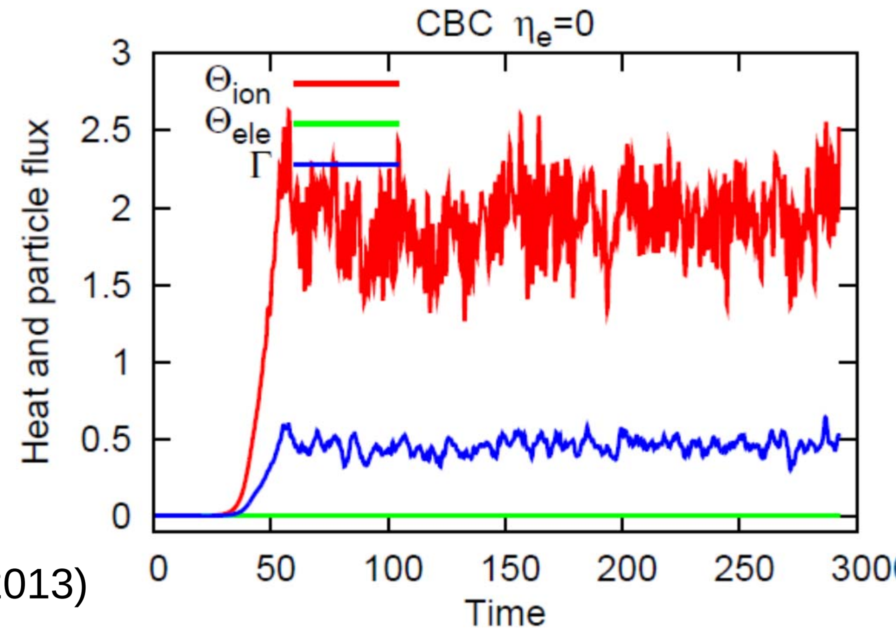
Small electron temperature gradient

$$\frac{1}{T_e}\frac{dT_e}{dr} << 1$$

- ETG is stabilized
- Saturated state is obtained.







A. Ishizawa, et.al., Nuclear Fusion, (2013)

Entropy transfer

- We can study the saturation mechanism of turbulence based on the conservation of quadratic quantities (entropy balance).

$$\frac{d}{dt}\left(\sum_s \delta S_{s,k} + W_{es,k} + W_{em,k}\right) = \sum_s \left(T_{s,k} + \frac{\Theta_{s,k}}{L_{Ts}} + \frac{T_s \Gamma_{s,k}}{L_{ps}} + D_{s,k}\right)$$

$$T_{s,k} = \sum_{k',k''} T_s(\mathrm{k};\mathrm{k}',\mathrm{k}'')$$

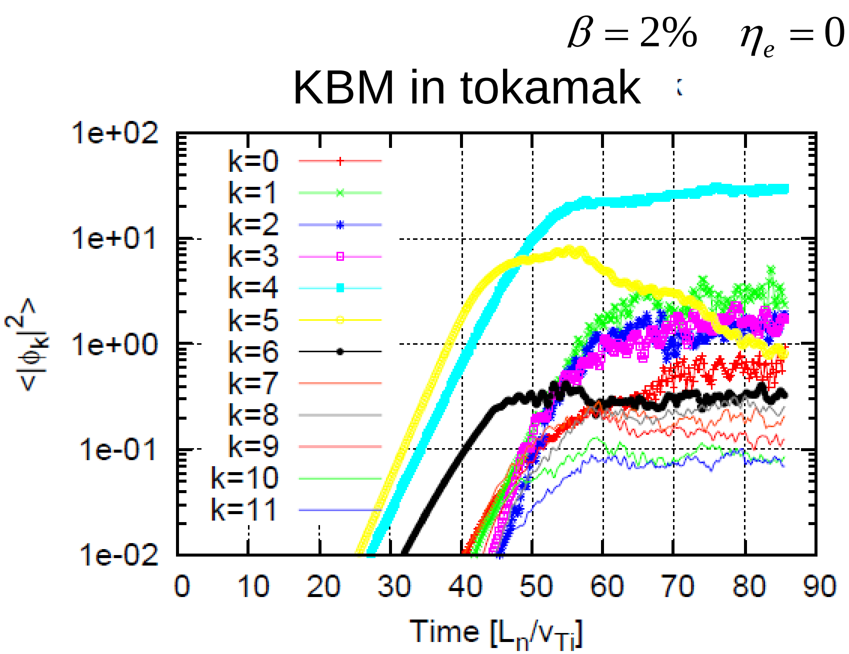
Entropy transfer function

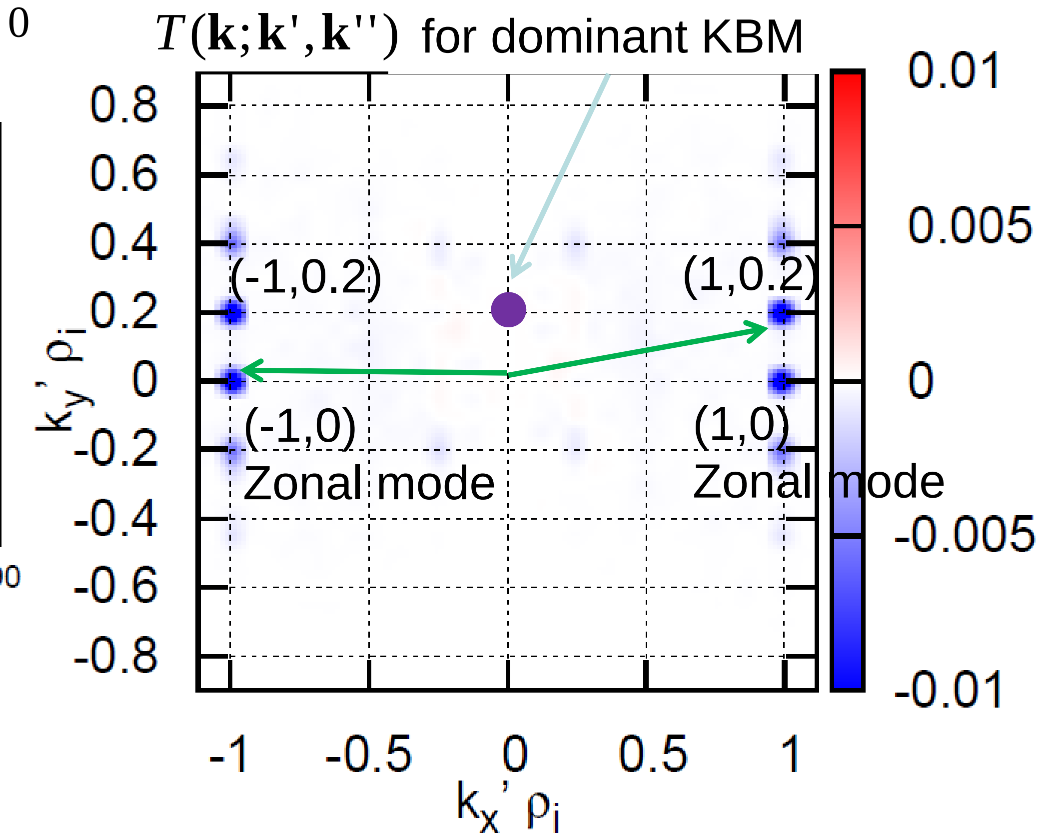
$$T(\mathbf{k};\mathbf{k}',\mathbf{k}'') = \left\langle \int dv^3 \frac{T_s h_{sk}^*}{2F_{Ms}} \delta_{k,k'+k''} \mathbf{b}\cdot\mathbf{k}'\times\mathbf{k}''(\chi_{sk'} h_{sk''} - h_{sk'}\chi_{sk''}) \right\rangle$$

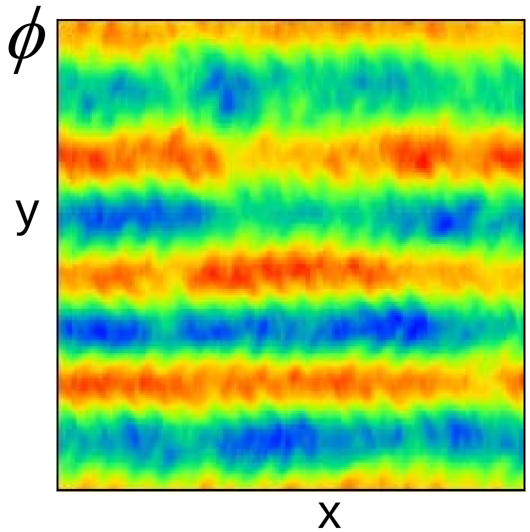
$$h_{sk} = f_{sk} + \frac{q_s}{T_s}\phi_k J_{0s} F_{Ms} \qquad \chi_{sk} = (\phi_k - v_{Ts} v_{//} A_{//k}) J_{0s}$$

$$T(\mathrm{k};\mathrm{k}',\mathrm{k}'') + T(\mathrm{k}';\mathrm{k}'',\mathrm{k}) + T(\mathrm{k}'';\mathrm{k},\mathrm{k}') = 0$$

Saturation mechanism of KBM turbulence

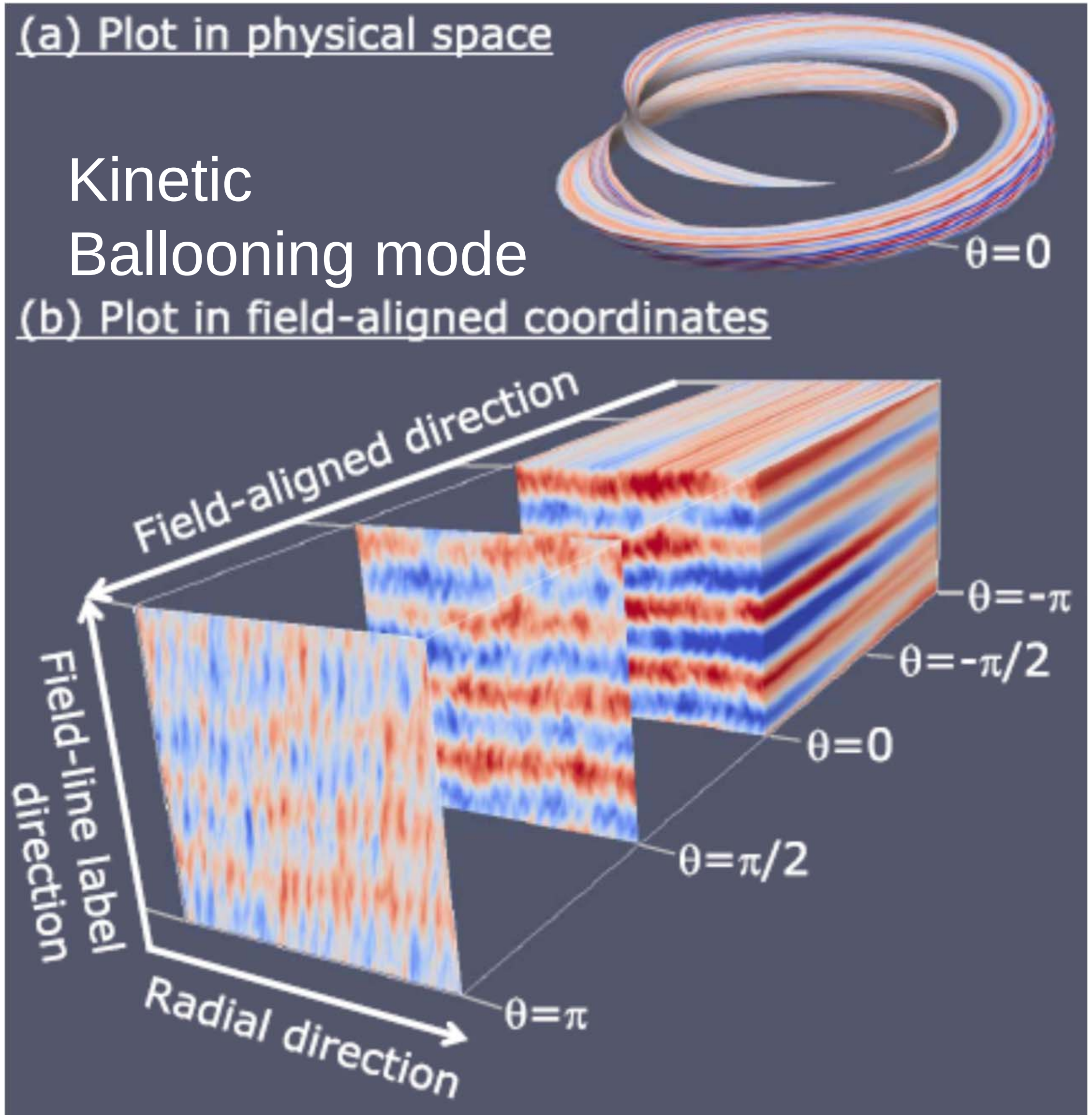






- The dominant KBM $(k_x, k_y) = (0, 0.2)$ is saturated bv nonlinear interaction with $(k_x, k_y) = (1., 0.2)$ connecting to $(k_x, k_y) = (0, 0.2)$
- Twisted modes appear along the field line and cause saturation of the KBM.

Saturation of KBM in a tokamak

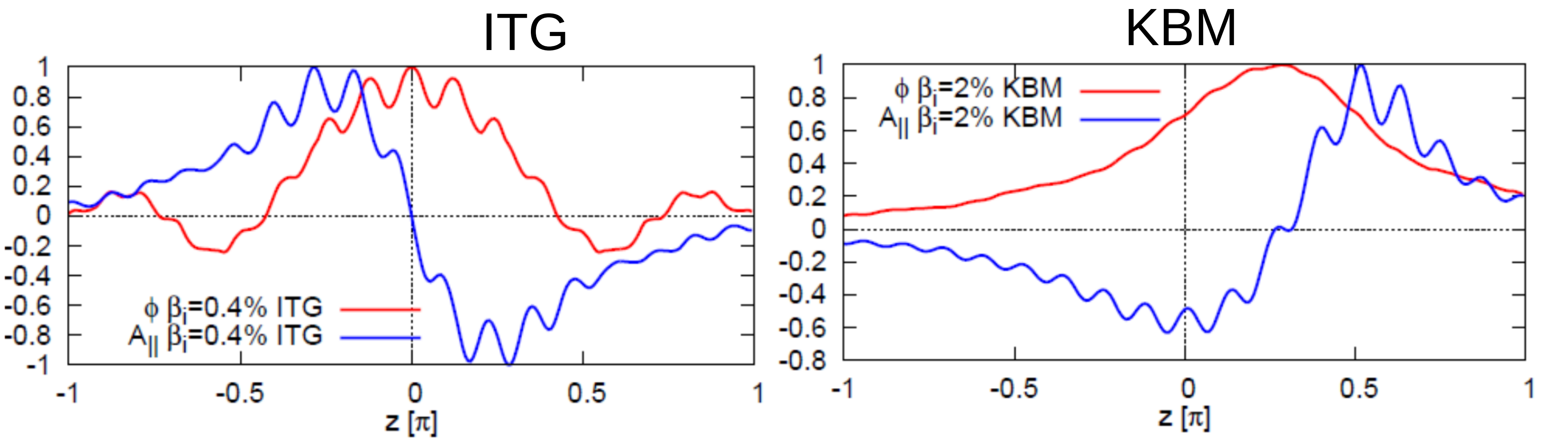


S. Maeyama, et.al., Phys. Plasmas, (2014)

- Twisted modes appear along the field line and cause saturation of the KBM.
- Mode structure along the magnetic field line play an important role in the saturation.

Saturation of KBM turbulence in helical plasmas

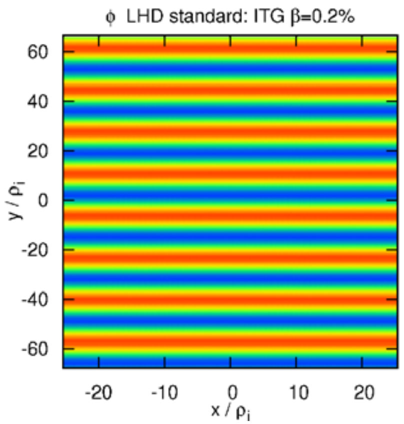
The most unstable KBM has finite radial mode number (finite theta_k)

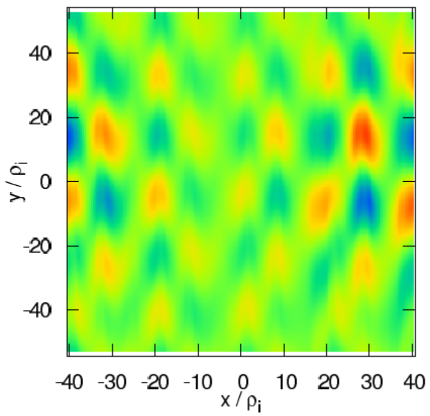


- The peak of the amplitude of electrostatic potential appears at finite z.

$$\theta_k = -k_x/(k_y \hat{s})$$

ϕ

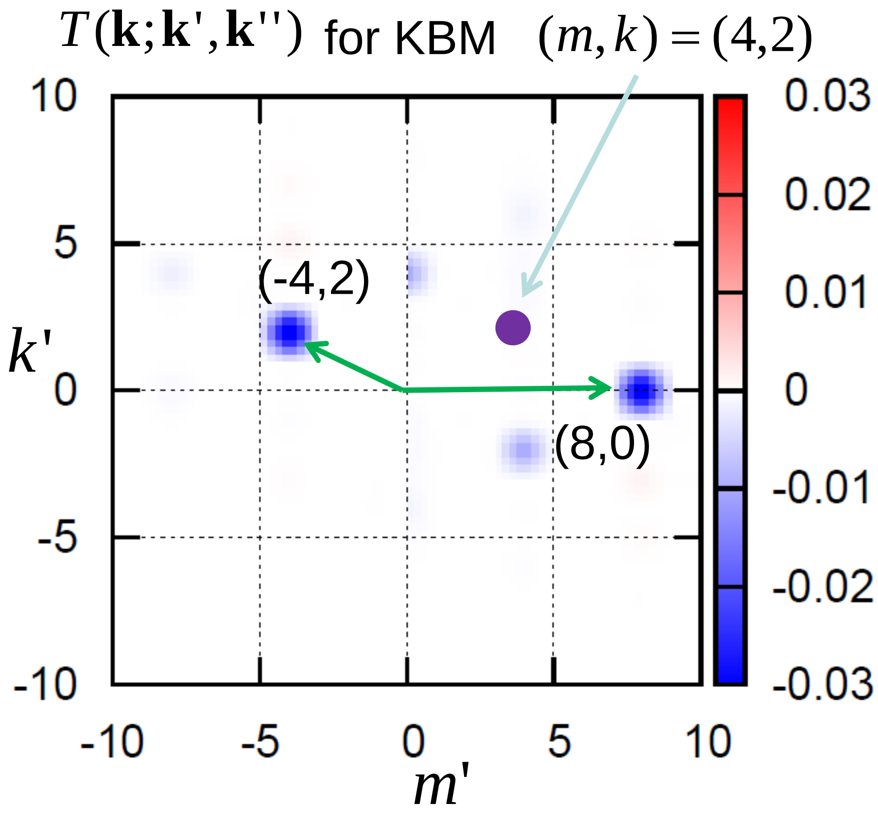




A. Ishizawa, et.al., Nuclear Fusion, (2013)

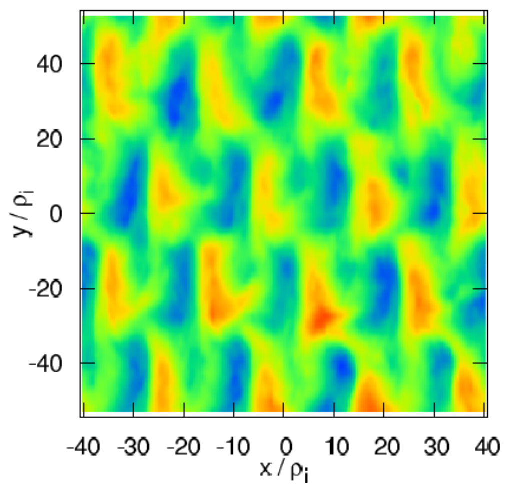
Saturation mechanism of KBM

- Turbulence is saturated by nonlinear interactions of oppositely inclined convection cells through mutual shearing.
  - The entropy of KBM with (m,k)=(4,2) is reduced by nonlinear interactions with (m',k')=(-4,2) and (m'',k'')=(8,0) modes.



$$(k_x,k_y)=(k_{x\min}m,k_{y\min}k)$$

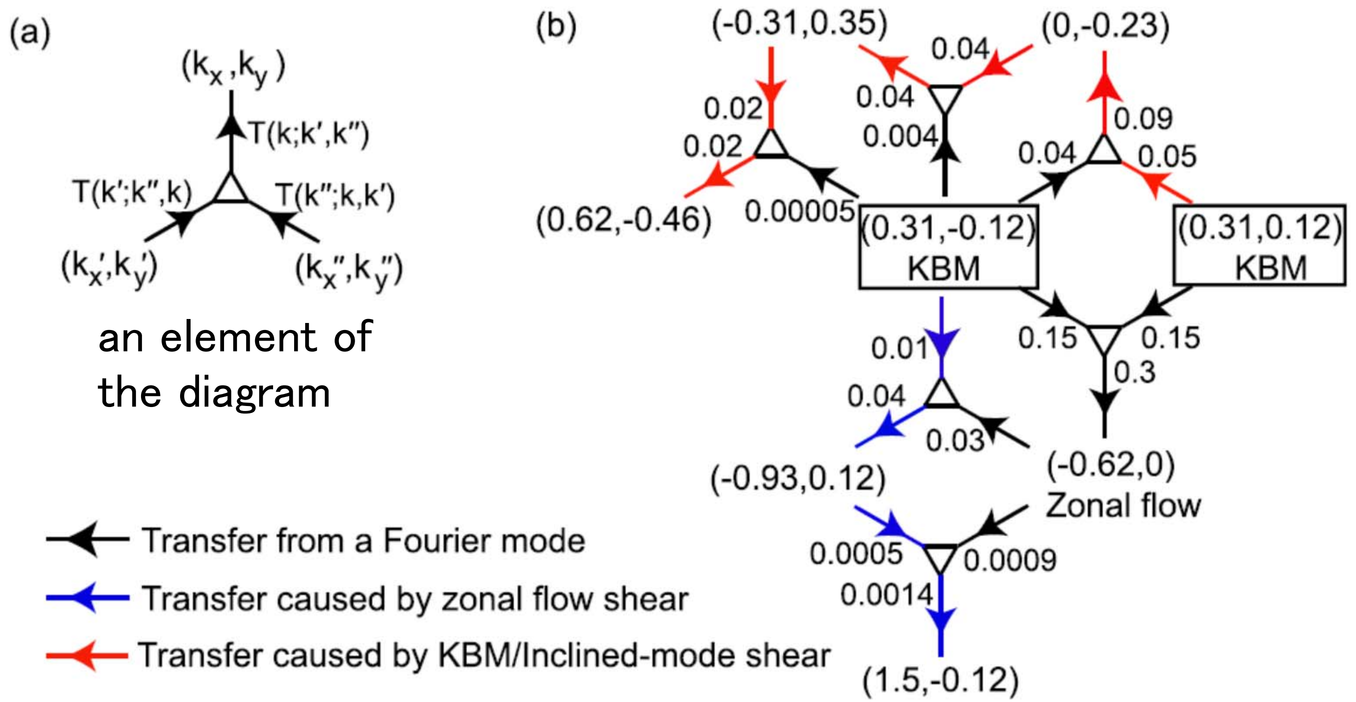
$$(k_{x\min}\rho_i,k_{y\min}\rho_i)=(0.077,0.058)$$



A. Ishizawa, et.al., Phys. Plasmas (2014)

Saturation mechanism I

- Diagram of nonlinear entropy transfer in the Fourier space
- Saturation of the KBM turbulence is caused by nonlinear interactions between dominant unstable modes with finite radial wavenumbers



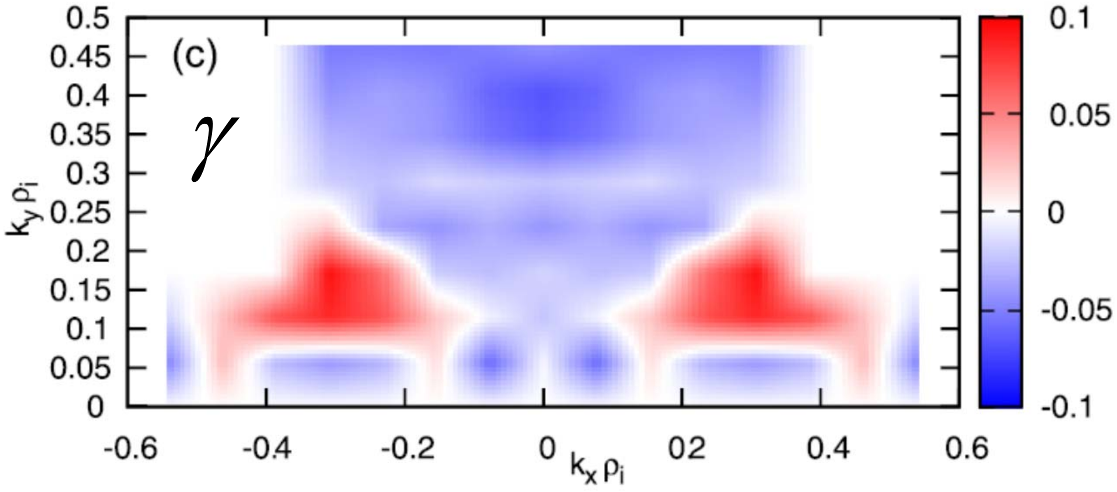
Entropy transfer function

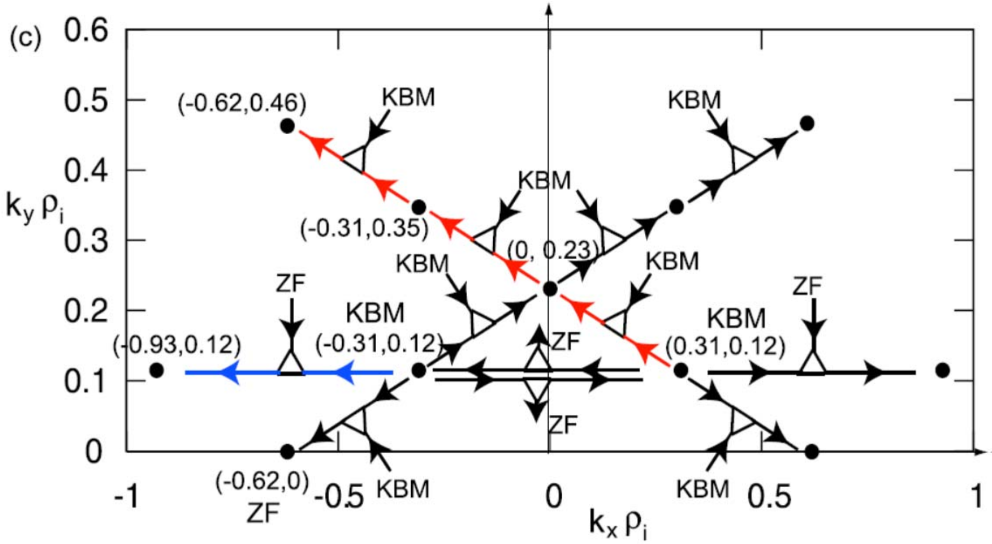
$$T(\mathbf{k};\mathbf{k}',\mathbf{k}'') = \left\langle \int dv^3 \frac{T_s h_{sk}^*}{2F_{Ms}} \delta_{k,k'+k''} \mathbf{b}\cdot\mathbf{k}'\times\mathbf{k}''(\chi_{sk'} h_{sk''} - h_{sk'}\chi_{sk''}) \right\rangle$$

$$h_{sk} = f_{sk} + \frac{q_s}{T_s}\phi_k J_{0s} F_{Ms}, \quad \chi_{sk} = (\phi_k - v_{Ts} v_{//} A_{//k}) J_{0s}$$

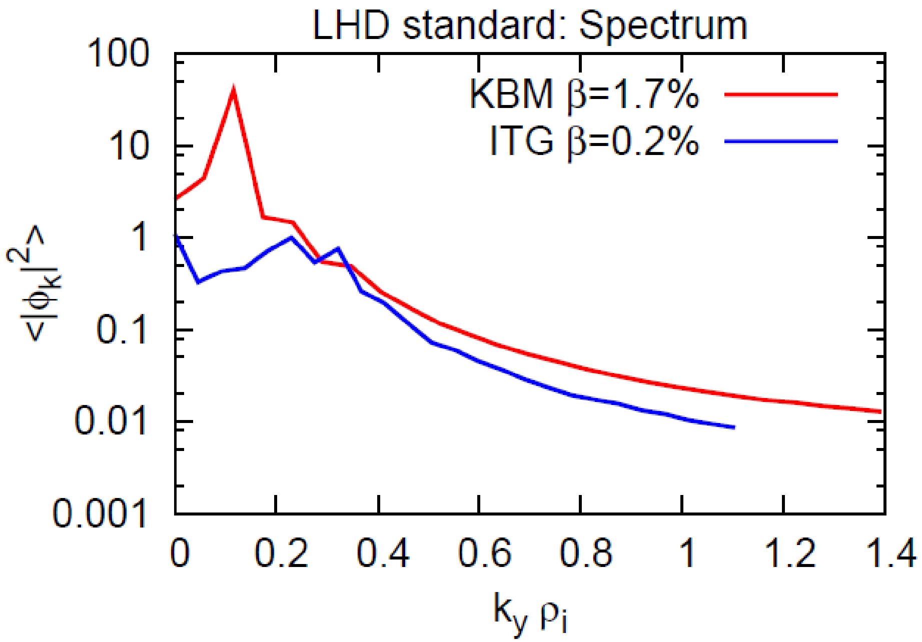
Saturation mechanism II

- The dominant KBM causes the transfer in the inclined direction and subsequently transforms the entropy from the other dominant KBM to higher Fourier modes, which are linearly stable.
- Hence, the growth of KBM is saturated by the nonlinear interactions of oppositely inclined convection cells through mutual shearing.





KBM turbulence is not efficient in the transport compared with ITG (Helical)



Energy flux

ITG $Q_i \approx 5 n_0 T_i v_{Ti} \rho_i^2 / L_n^2$

KBM $Q_i \approx 3 n_0 T_i v_{Ti} \rho_i^2 / L_n^2$

- Zonal flow of KBM turbulence is much weaker than that of ITG turbulence.

A. Ishizawa, et.al., Phys. Plasmas (2014)

Summary

- Electromagnetic gyrokinetic simulation enables us to study turbulent transport in finite beta torus plasmas.
- Conserved quantities
  - Quadratic conserved quantity (Entropy variable)
    - Entropy transfer in the Fourier space
  - Parity symmetry along the field line (Linear)
    - Ballooning parity: ITG, TEM, ETG, KBM
    - Tearing parity: MTM
- Nonlinear simulations
  - Beta dependence of turbulent transport
  - Saturation problem of turbulence at finite-beta
  - Turbulence at high-beta with small electron temperature gradient