Computational kinetic MHD in 2D and 3D systems

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Outline

- Introduction
- fully gyro-kinetic appoach in 2D
- What is kinetic MHD?
- Then, why we build a kinetic MHD model at all?
- Alfvén waves in stellarators
- numerical 3D kinetic MHD model: CAS3D-K
- local analytic kinetic MHD model
- application: stability boundaries for TAE in W7-AS/W7-X
- extentions of the model: CKA/EUTERPE, AE3D, VENUS

Introduction

kinetic effects may interact with ideal MHD modes:

- destabilization of MHD gap modes by resonant interaction of fast particles
- source of free energy: density or temperature gradient of fast particles
- experimental observations in stellarators (see lecture of K. Toi):
  - W7-AS: Weller et al. (1998, 2000, 2003)
  - CHS/LHD: Toi et al. (2000, 2004)
  - HSX (Brower et al. 2006)
- this lecture will cover linear physics
  for a non-linear approach see lecture of Y. Todo

Proper attac to resonance phenomena:

fully kinetic approach - PIC code (GYGLES)

- Global linear 2D fully gyrokinetic δf code
- Slab, pinch and tokamak geometries are available
- The code solves linearized gyrokinetic Vlasov-Maxwell system

$$\frac{\partial}{\partial t}\delta f + \{\delta f, H_0\} = -\left\{F_0, e\left\langle \phi - v_{\parallel} A_{\parallel}\right\rangle\right\}$$

$$n_i = n_e \ , \quad -\nabla_{\perp}^2 A_{\parallel} = \mu_0 (j_{\parallel i} + j_{\parallel e})$$

- The "Klimontovich" representation for the distribution function

$$\delta f = e^{iS(\vec{x})} \sum_{\nu=1}^{N_p} w_\nu \delta(z - z_\nu)$$

- The "Ritz-Galerkin" representation for the fields (using B splines)

$$\phi(\vec{x}) = e^{iS(\vec{x})} \sum_{k=1}^{N_{\mathrm{FE}}} \phi_k \Lambda_k(\vec{x}) \ , \quad A_{\parallel}(\vec{x}) = e^{iS(\vec{x})} \sum_{k=1}^{N_{\mathrm{FE}}} a_k \Lambda_k(\vec{x}) \ ,$$

cancellation problem

parallel Ampére's law:

$$\left(\frac{\beta_i}{\rho_i^2}+\frac{\beta_e}{\rho_e^2}+\frac{\beta_f}{\rho_f^2}-\nabla_\perp^2\right)A_\parallel=\mu_0\left(\bar{j}_{\parallel i}+\bar{j}_{\parallel e}+\bar{j}_{\parallel f}\right)$$

gyro-center current:

$$\bar{j}_{\parallel s}=q_s\int \mathrm{d}^6Z\,\delta f_s\,v_\parallel\,\delta(\mathrm{R}+\rho-\mathrm{x})$$

thermal gyro-radius: $\rho_s=\sqrt{m_sT_s}/(eB)$, **plasma beta:** $\beta_s=\mu_0n_0T_s/B_0^2$

parts of $\bar{j}_{\parallel e}$ (noisy) have to cancel analytic $\beta_e/\rho_e^2\gg k_\perp^2$

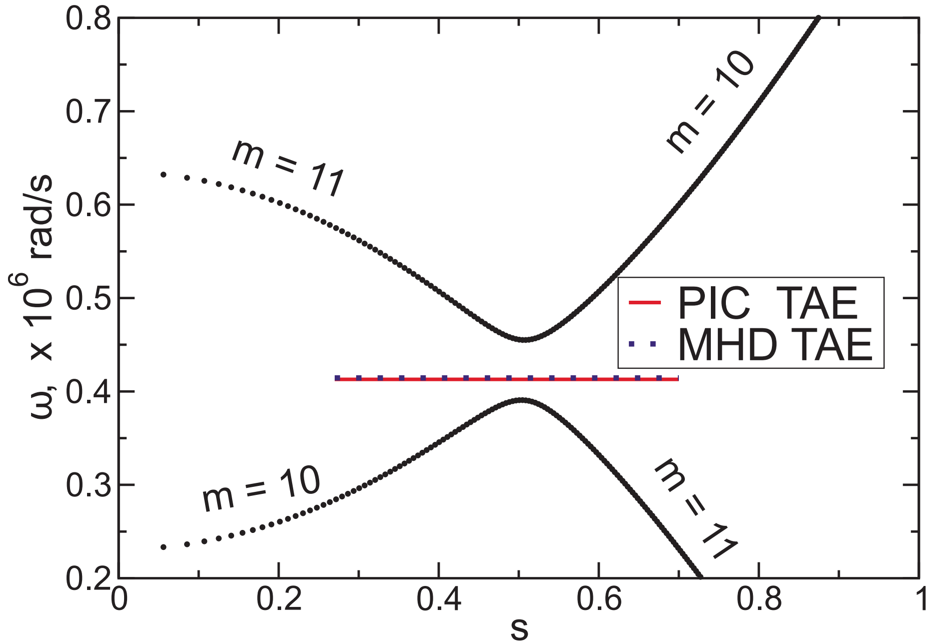
details of numerical algorithm:

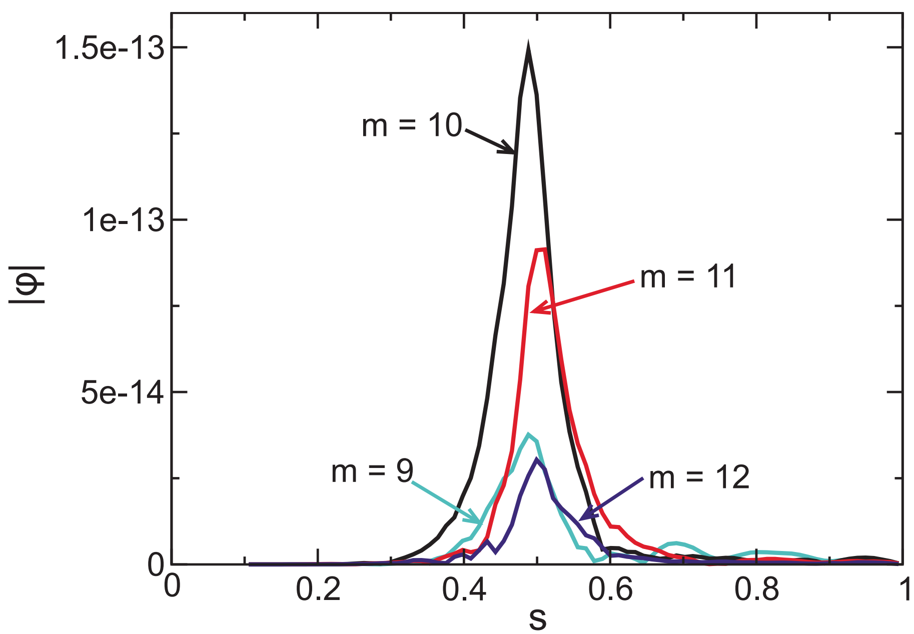
R. Hatzky, A. Könies, and A. Mishchenko, J. Comp. Phys. 255, 568 (2007)

problem gets harder with small mode numbers and small aspect ratio (coupling)

problem (up to JET size) solved last year

Electro-magnetic gyro-kinetic PIC simulation (tokamak)

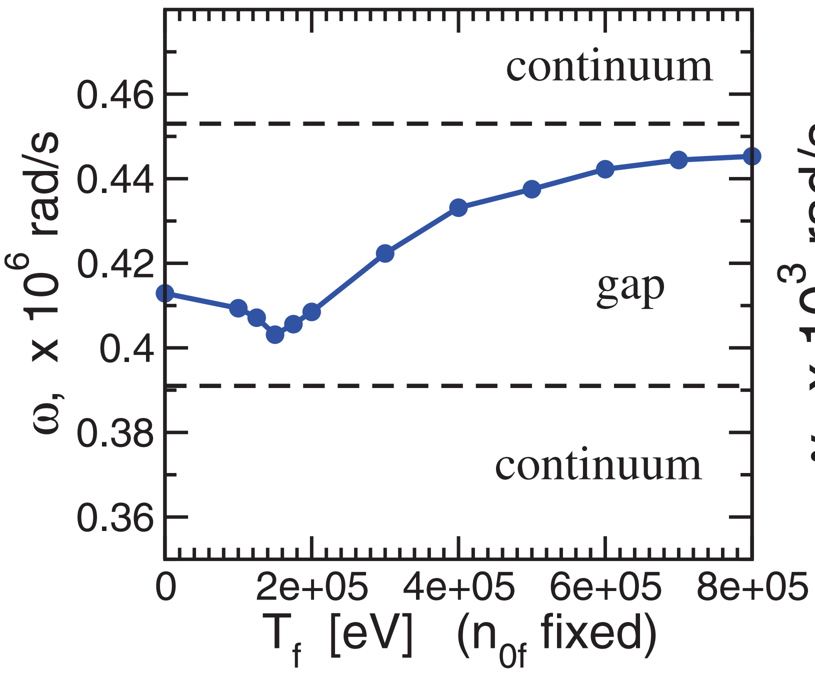


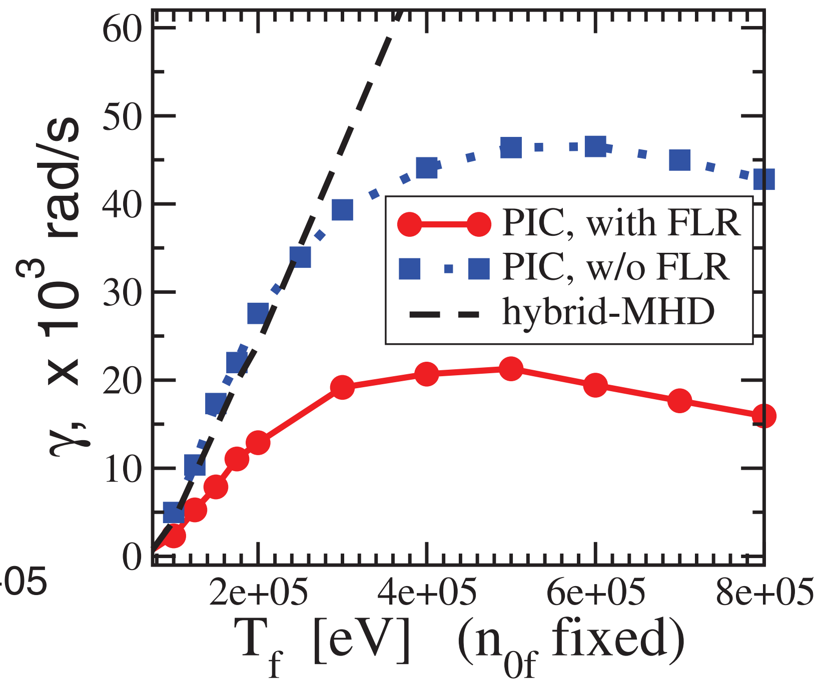


Toroidal Alfven Eigenmode in a tokamak (A=10) (GK PIC against MHD)

satisfying agreement with MHD frequency, mode structure in qualitative agreement with MHD (no fast particles)
(Mishchenko, Könies, Hatzky , PoP, 2009)

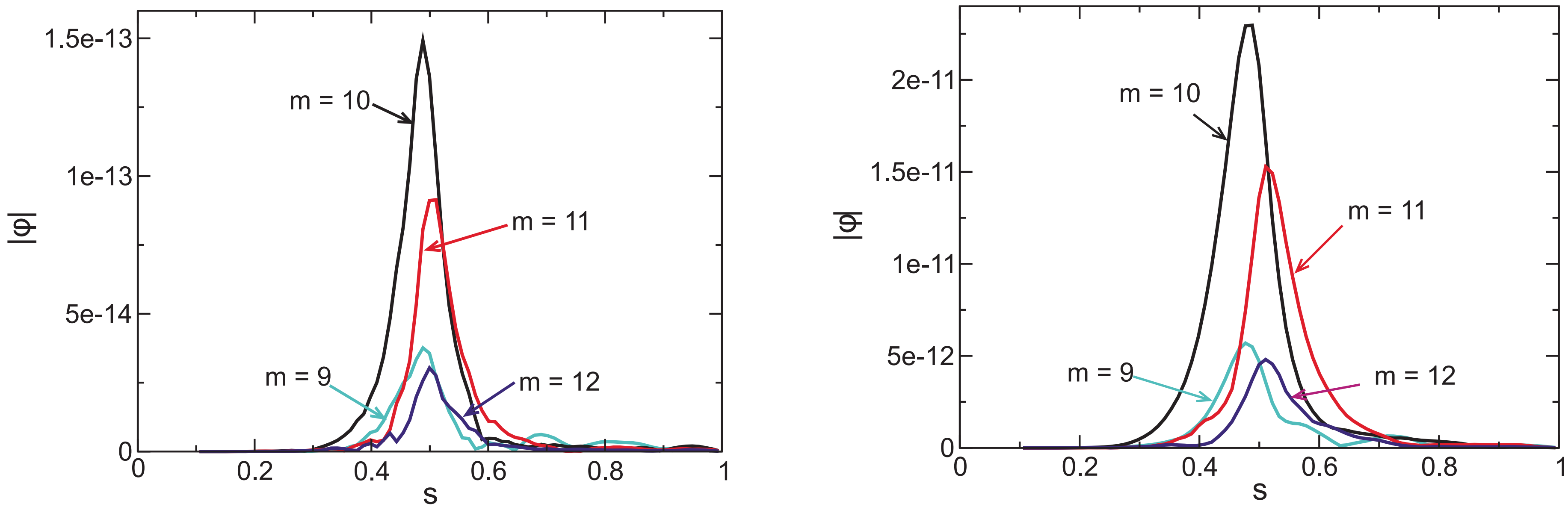
TAE from gyro-kinetic PIC simulation (tokamak)





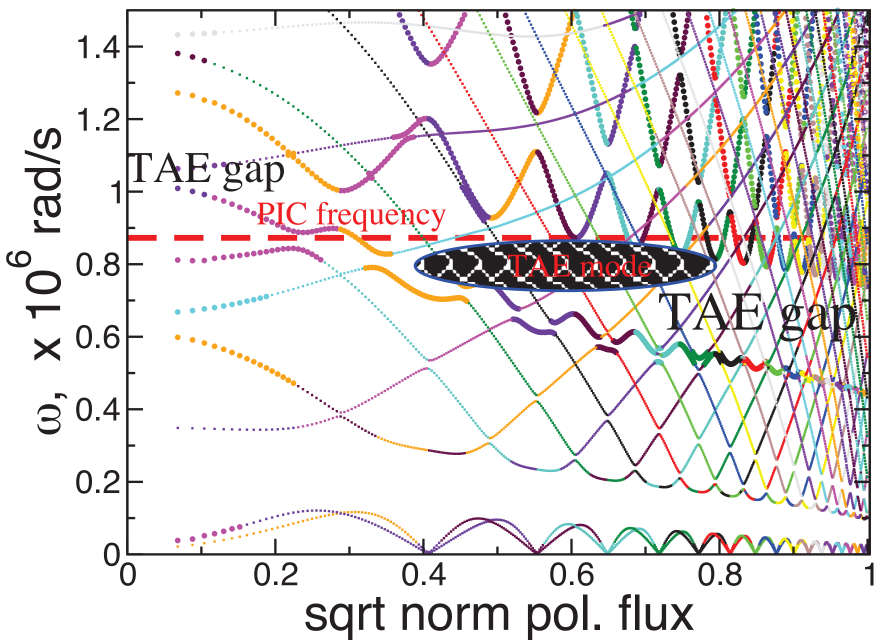
temperature dependence of frequency and growth rate for fast particles with a Maxwellian distribution

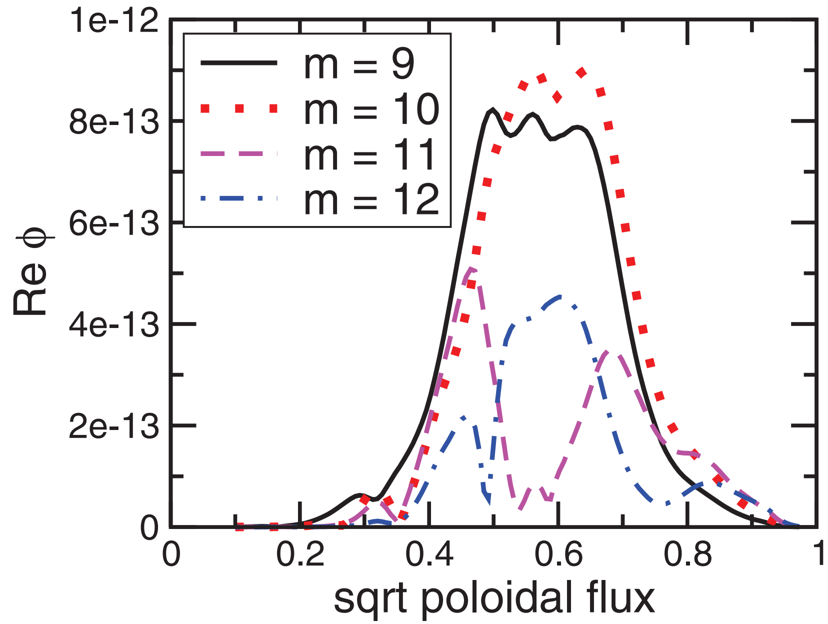
TAE from gyro-kinetic PIC simulation (tokamak)



comparison of mode structure for different fast particle energies:
no fast particles (left) $T_f = 500\text{eV}$ **(right)**
only slight changes in mode structure (perturbative approach possible)

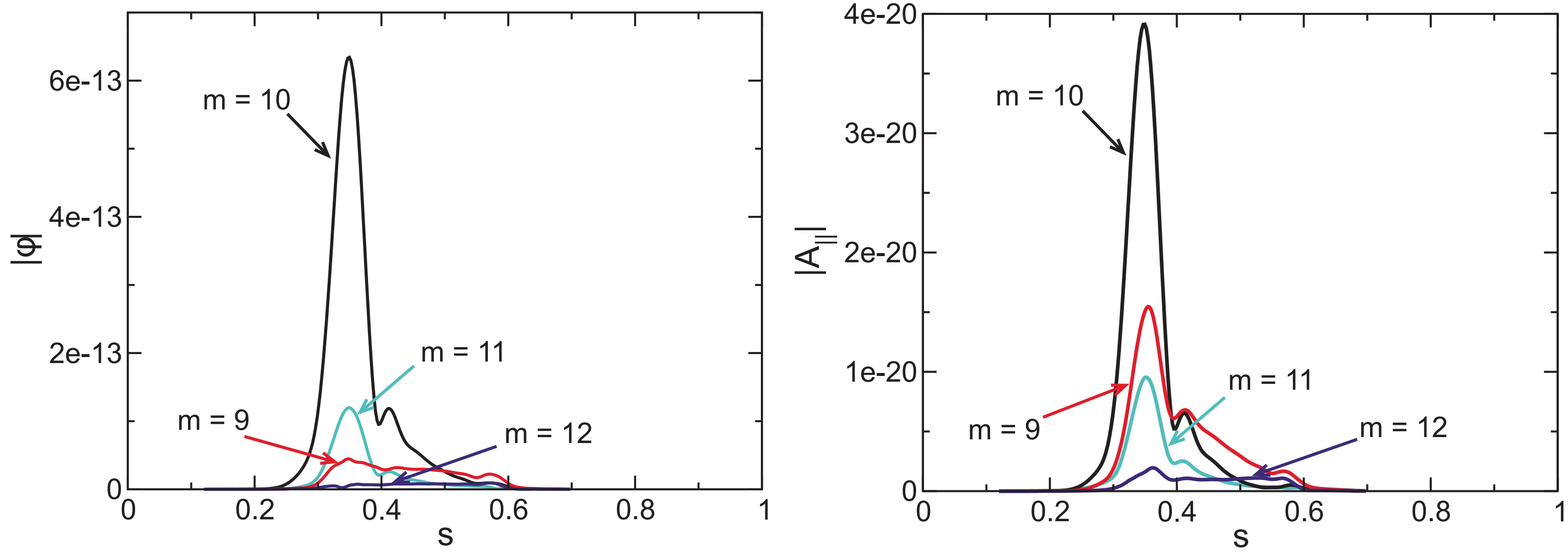
TAE from gyro-kinetic PIC simulation (tokamak)





$A = 3$ circular tokamak, JET parameters ($B = 3.45$T)
mode is still located in the gap if fast ion pressure ($\beta_f \approx 4.7\%$) is also considered, TAE found shows signs of radiative damping
no transition to EPM (Mishchenko, Könies, Hatzky , PoP, 2011)

EPM from gyro-kinetic PIC simulation (tokamak)



$A = 3$ circular tokamak, JET parameters ($B = 3.45$T)

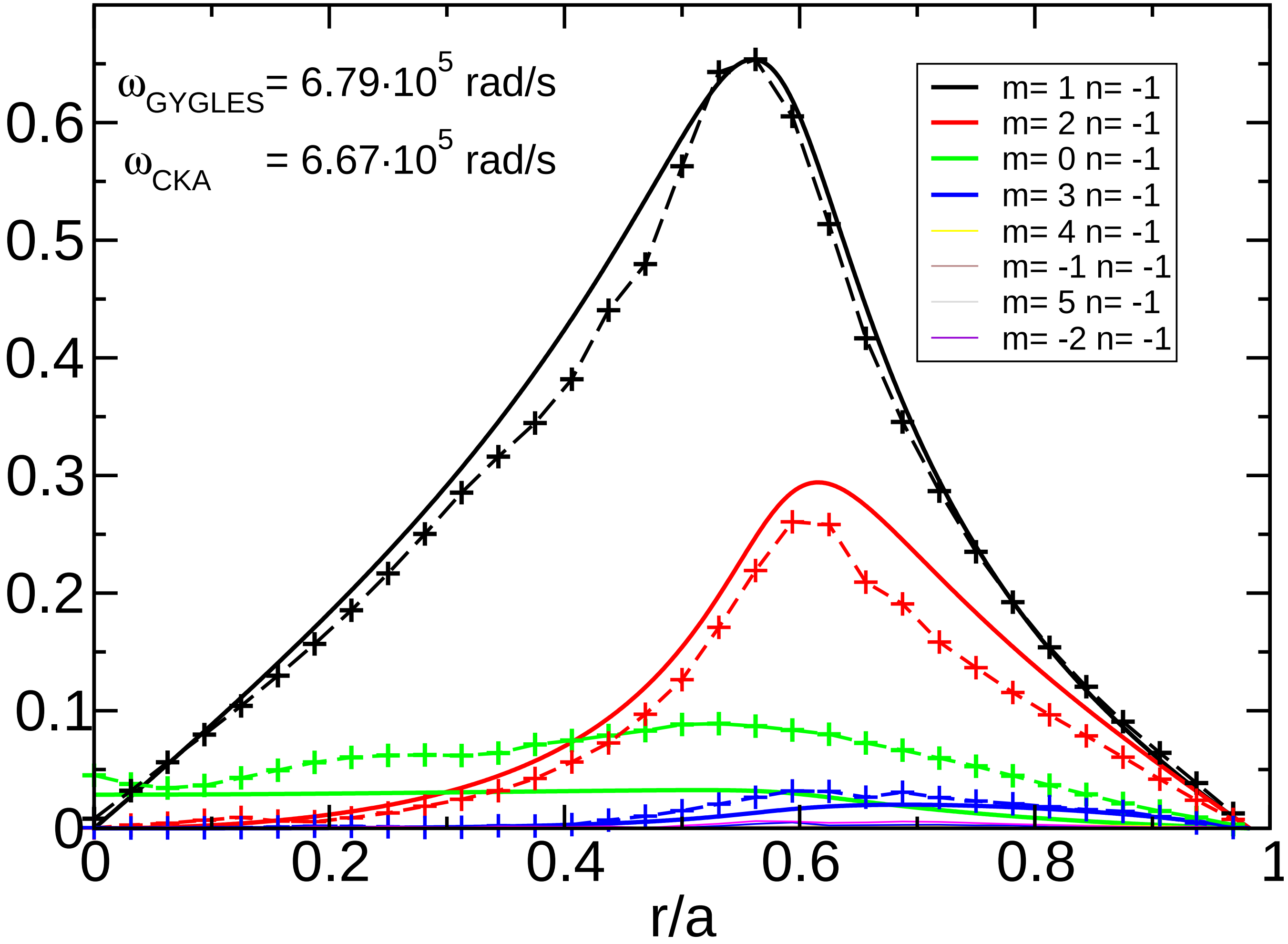
($\beta_f \approx 0.47\%$ i.e. $\beta_f/\beta = 0.75$) but $T_f = 50$keV and $n_f = 6.0 \cdot 10^{17}$m^{-3}
energetic particle mode

(Mishchenko, Könies, Hatzky , PoP, 2009)

$(n = 1)$-TAE

full lines: reduced MHD (CKA)

dashed lines with symbols: gyro-kinetic (GYGLES)



Kinetic MHD

. . . is a fluid-kinetic hybrid picture:

MHD description for the bulk plasma

gyro- kinetic description for the fast particle species

coupling: pressure or current term in MHD equations is supplemented by the fast particle pressure

(see e.g.
Gorelenkov, Cheng, Fu 1999
Park et al. PoP 1999)

this talk: linear kinetic MHD with perturbative calculation of the growth rate

Why should we use kinetic MHD?

counter arguments:

- Alfvén mode physics actually kinetic problem
- complete linear gyro-kinetic solution exists in 2D (GYGLES, LIGKA by Ph. Lauber)

but:

- for 3D linear gyro-kinetics PIC code (EUTERPE) available, but no mode example (current research)
- question of effort:
  2D calculations of Alfvén modes: 1-2 days, 32-128 Processors, from 2 Mill. particles
  3D electromagnetic calculations of ITG modes: 2-3 weeks on 128 proc., 16 Mill. particles + human assistance (R. Hatzky, pers. comm.)
- non-linear physics, mode saturation
- still applicable in many cases (cf. lecture of N. Gorelenkov)

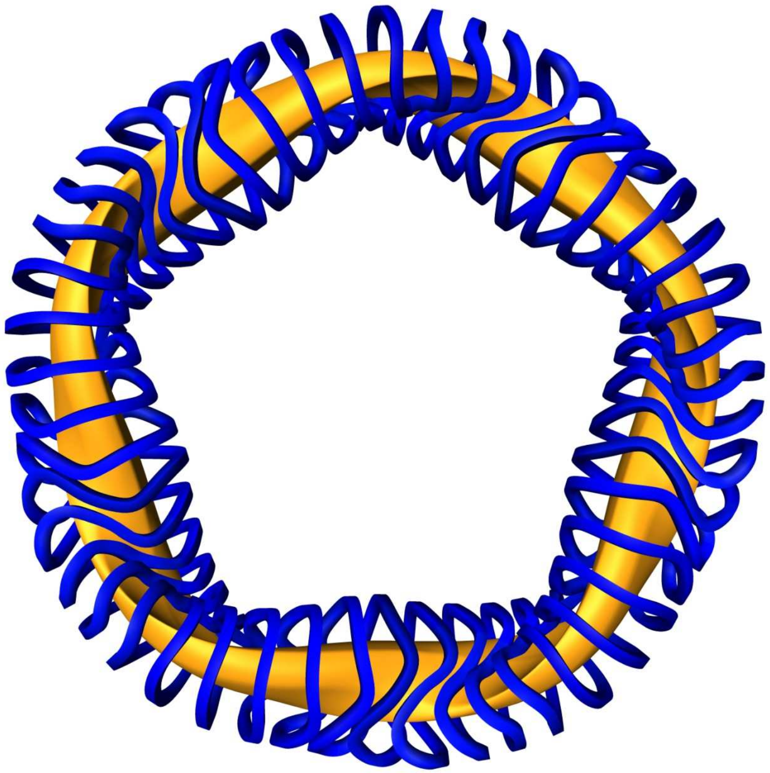
stable MHD spectrum revisited

- looking for particle interaction with the stable part of MHD spectrum (at least in most of the cases)
- stable MHD spectrum:
  analogy to Schrödinger equation in a solid state

MHD	solid state
$\omega^2 W_{kin}(\xi^*, \xi) = W_{mag}(\xi^*, \xi)$	$E\|\Psi\|^2 =< \Psi^*\|H\|\Psi >$
slab/cylinder dispersion relation:	free electron model:
$\omega^2 = k_{\|\|}^2 v_A^2$	$E = \hbar^2 k^2/(2m^*)$
degeneracy removed by symmetry breaking terms $\Rightarrow$ gap	degeneracy removed by potential $\Rightarrow$ gap

difference: MHD allows for global modes in the gap

3D linear modes: mode families



example W7-X: $N_P = 5$

stellarators are periodic:
number of field periods: N_P
$0 \leq \varphi \leq 2\pi$
Bloch theorem applies to modes:

$$\phi(\vec{r}) = e^{iN\varphi} \sum_{mk} e^{i(m\theta + kN_P\varphi)} \phi_{mk}(r)$$

comparing with

$$\phi(\vec{r}) = \sum_{mn} e^{i(m\theta + n\varphi)} \phi_{mn}(r)$$

it follows:
only modes having

$$n \equiv N (\bmod N_P)$$

can couple $\Rightarrow$ **mode families**

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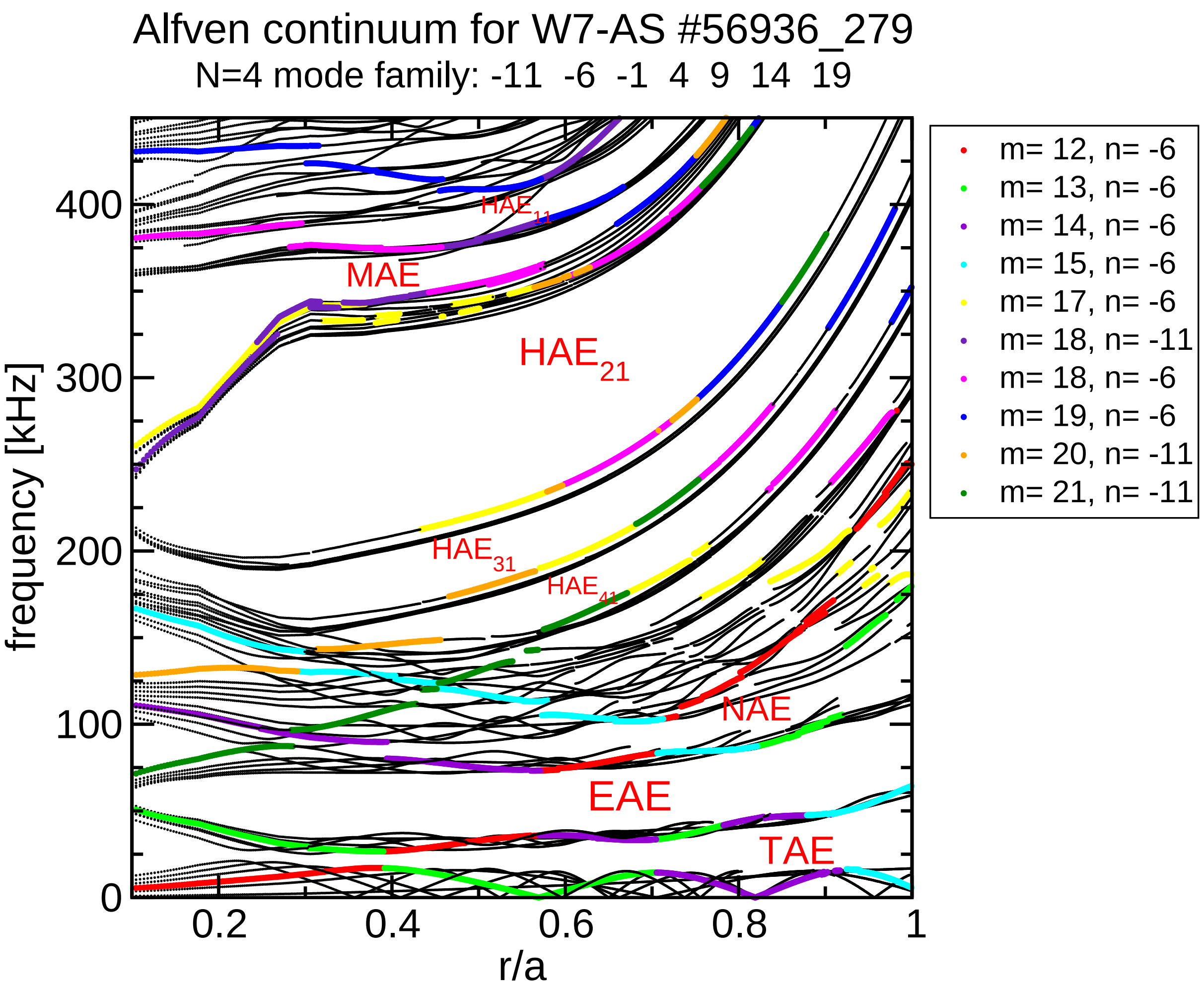
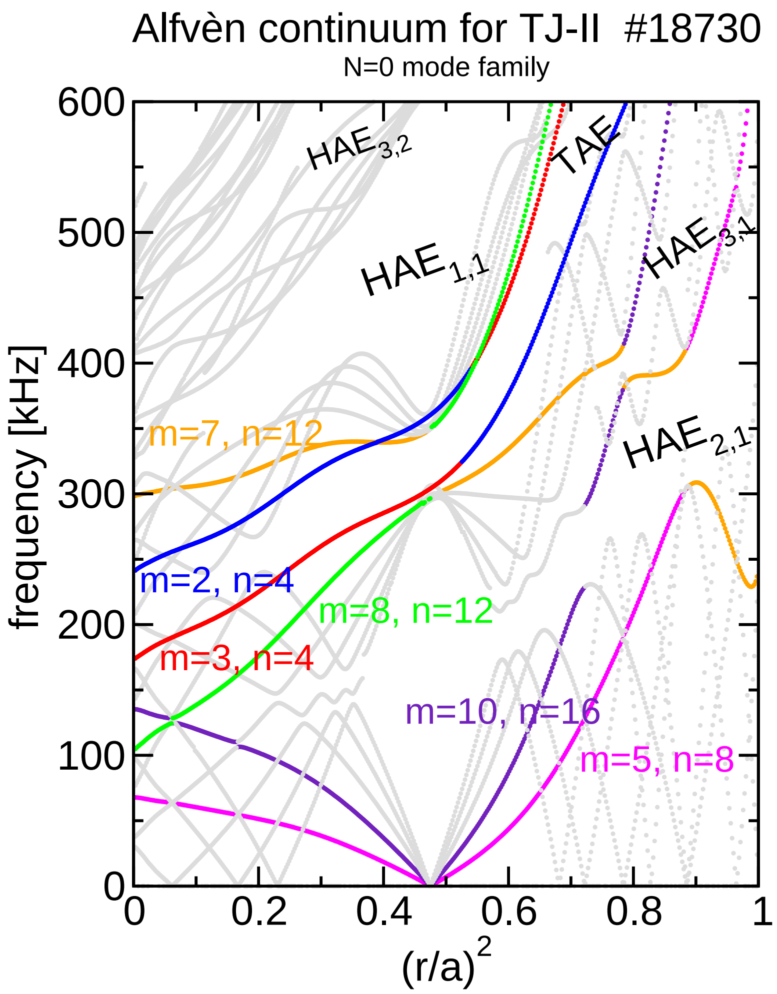
3D ideal MHD continuum: W7-AS and TJ-II





Kinetic MHD

1. restriction to MHD-like perturbations
   $\phi = 0$ no electrostatic potential
   $\vec{B}^{(1)} = \vec{\nabla} \times \left(\vec{\xi} \times \vec{B}\right)$
   $\vec{A}^{(1)} = \vec{\xi} \times \vec{B}$

2. derivation of an energy functional from the MHD moment equation

$$\vec{\nabla} \cdot \overleftrightarrow{P} = -\vec{B} \times \left(\vec{\nabla} \times \vec{B}\right)$$

3. replace $\overleftrightarrow{P}$ with a kinetic expression, i.e. an expression involving integrals of the distribution function

remark: this is equivalent to calculate growth/ damping rates considering the particle-wave energy transfer

Drift kinetic equation

Vlasov equation after transformation to guiding center variables and averaging over the gyro phase:
(e.g. Porcelli et al. 1994, Catto et al. 1980, Littlejohn 1983, cf. Hahm 1988)

$$\frac{\partial f}{\partial t} + \dot{\vec{R}} \cdot \frac{\partial f}{\partial \vec{R}} + \dot{v}_\parallel \frac{\partial f}{\partial v_\parallel} + \dot{y} \frac{\partial f}{\partial y} = 0$$

$y = \mu B$: perpendicular energy, $v_\parallel$: parallel velocity $\parallel \vec{B}$
$\vec{R}$: location of the guiding center

distribution function $f(\vec{R}, y, v_\parallel, t)$: distribution of guiding centers

correct up to first order in

$$\delta = \frac{\text{gyro radius}}{\text{system length}} \ll 1$$

Drift kinetic equation

Linearization: $f = F + f^{(1)}$
(equilibrium part: F **+ perturbation:** $f^{(1)}$ **)**

zero order:

$$\dot{\vec{R}}^{(0)} \cdot \left(\frac{\partial F(\epsilon, \mu, \vec{R})}{\partial \vec{R}} \right)_{\epsilon,\mu} = 0$$

- **regard this equation as being approximatively solved**
- **for times scales with negligible drifts:**

$$F = F(s, \epsilon, \mu, \sigma)$$

s: **flux label;** σ: **sign of** $v_{||}$

Drift kinetic equation

linearized 3D drift kinetic equation to <u>first order</u>:

- $f^{(1)}$ splits into an adiabatic ...

$$f^{(1)} = \vec{A}^{(1)} \cdot \frac{\vec{b} \times \vec{\nabla} F}{B} + Ze\phi^{(1)} \left(\frac{\partial F}{\partial \epsilon} \right)_{\vec{R},\mu} - \frac{B^{(1)}}{B} \left(\frac{\partial F}{\partial \mu} \right)_{\vec{R},\mu} + h^{(1)}$$

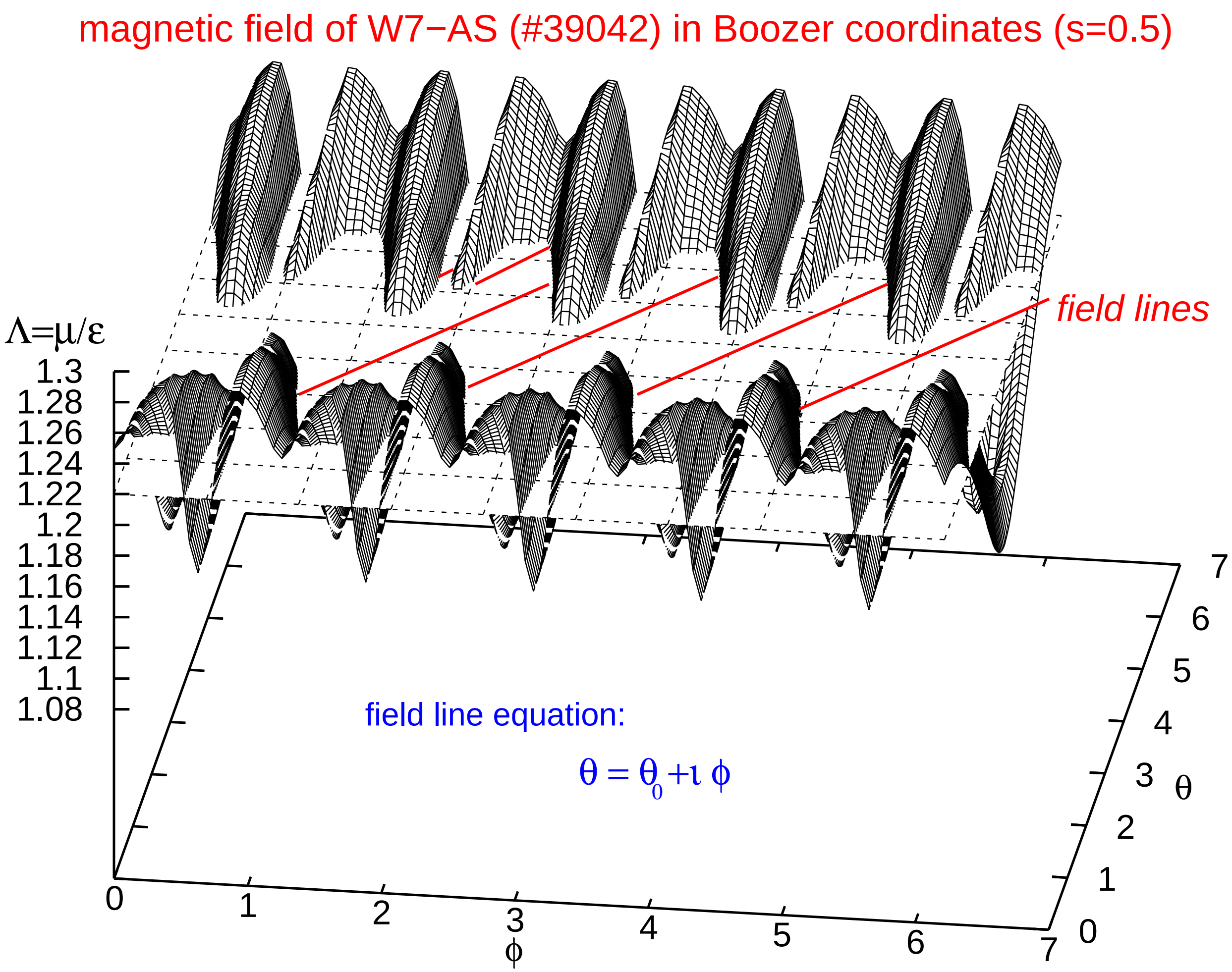
- ... and non-adiabatic part:

$$\frac{d}{dt} h^{(1)} = \left[\left(\frac{\partial F}{\partial \vec{R}} \right) \cdot \frac{\vec{b} \times \vec{\nabla}}{M\Omega} + \left(\frac{\partial F}{\partial \epsilon} \right) \frac{\partial}{\partial t} \right] L^{(1)}$$

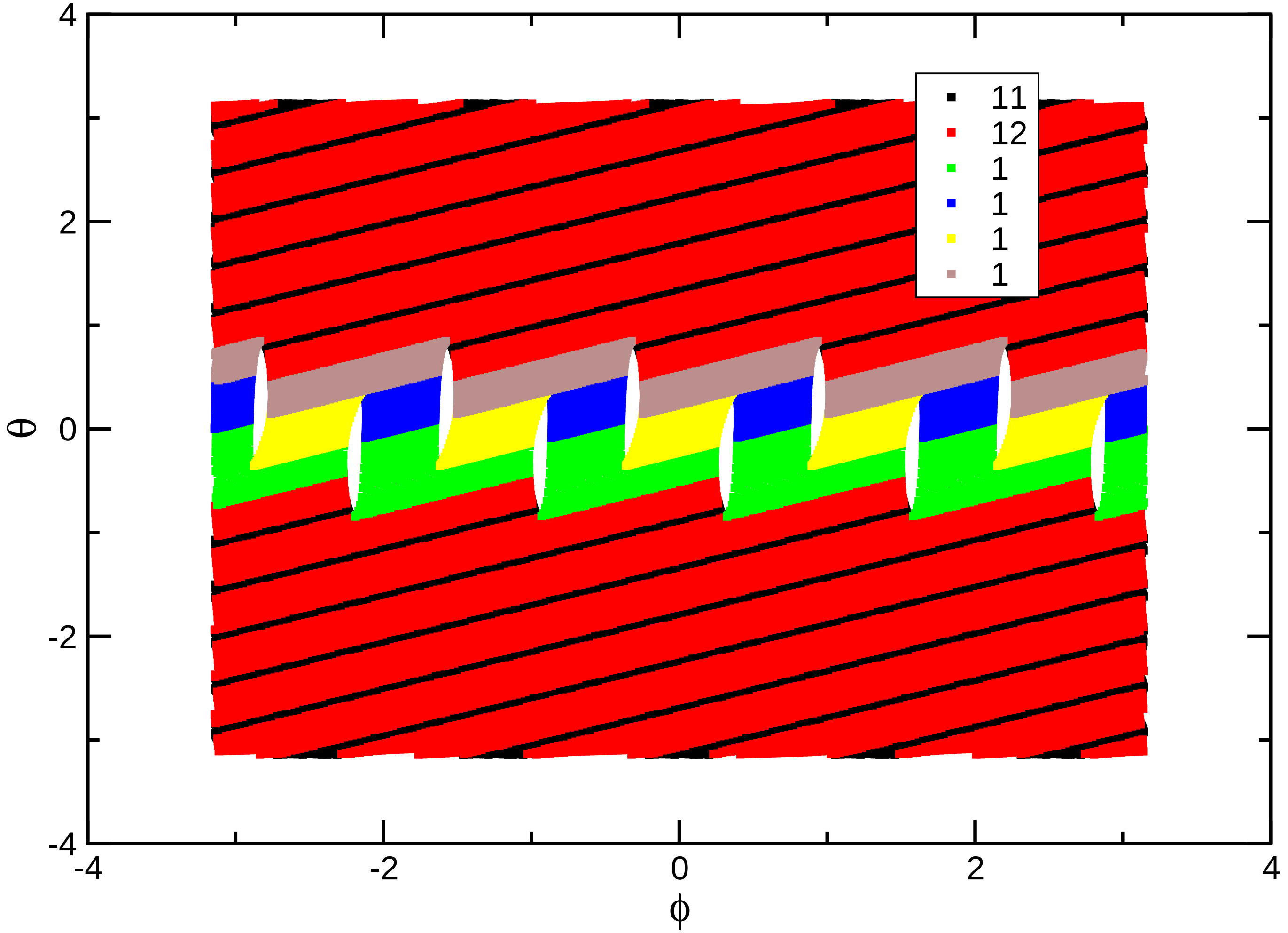
$L^{(1)}$: perturbed Lagrangian $L^{(1)} = \dot{\vec{R}} \cdot \vec{A}^{*(1)} - \mu B^{(1)} - Ze\phi^{(1)}$

integration along field lines
bounce averaged drifts within flux surface considered
no radial drifts

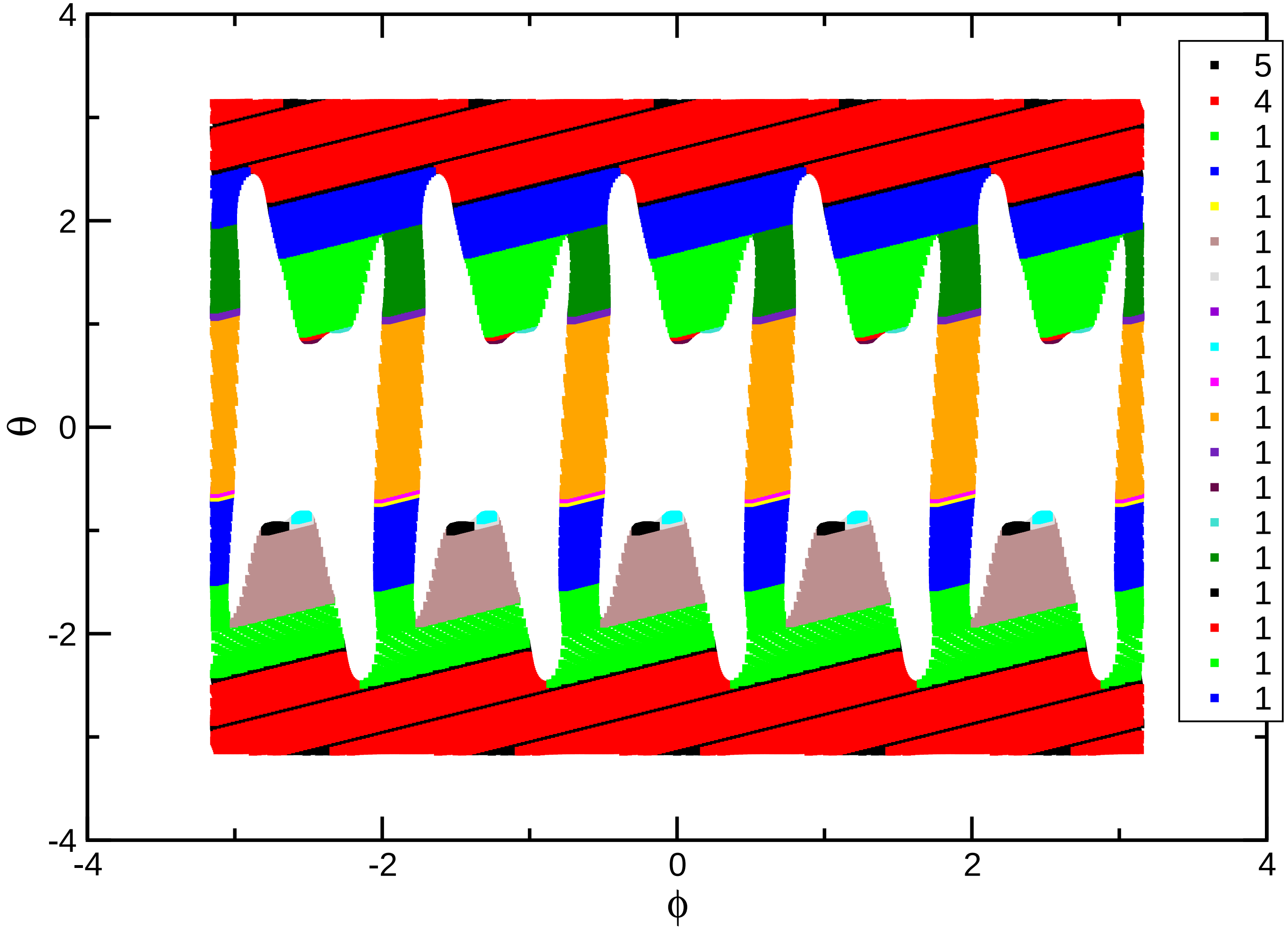
field line orbits (W7-AS)



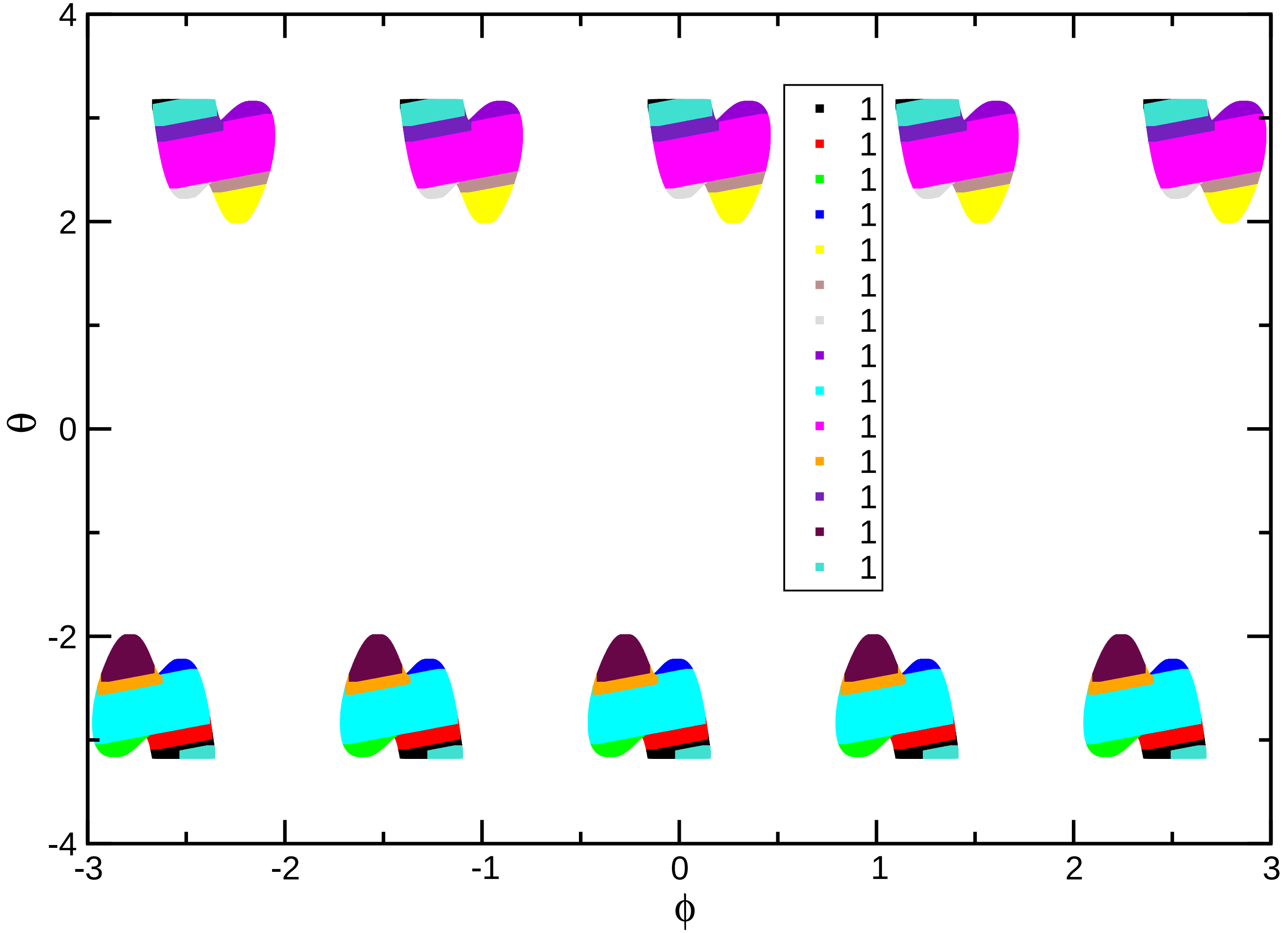
field line orbits (W7-AS)



field line orbits (W7-AS)



field line orbits (W7-AS)



Kinetic energy integral

there is an energy integral considering kinetic effects

$$\delta W_{\text{kin}} = \omega^2 \frac{1}{2} \int d^3\vec{x} \left| \vec{\xi}_\perp \right|^2 \rho_M = \delta W_{\text{mag}} + \sum_{s=\text{i,e,fast}} \delta W_s(\omega)$$

(Kruskal/Oberman 1958 ... Antonsen/Lee 1984)

$$\delta W_{\text{mag}} = \frac{1}{2} \int d^3\vec{x} \left\{ \left| B_\perp^{(1)} \right|^2 + \left| B_\parallel^{(1)} \right|^2 + \vec{j}_\parallel \cdot \left(\vec{\xi}_\perp \times \vec{B}_\perp^{(1)} \right) - \frac{B_\parallel^{(1)}}{B} \vec{\xi}_\perp^* \cdot \vec{\nabla} p + \left(\vec{\nabla} \cdot \vec{\xi}_\perp^* \right) \left(\vec{\xi}_\perp \cdot \vec{\nabla} p \right) \right\}$$

the non-adiabatic contributions from the hot and thermal component replace the MHD fluid compression term
the contributions from the thermal plasma ($\delta W_{\text{i,e}}$) and the fast particles δW_{fast}) depend on the perturbed particle Lagrangian $L^{(1)}$

(A. Könies, PoP 2000)

Kinetic contribution

particle- wave- energy- exchange by resonant interaction

$$\delta W_s = \frac{\pi}{M_s^2}\left\{\sum_\sigma\right\}\int ds\int d\varphi\int d\mu\, d\epsilon\left(-\int\frac{d\vartheta}{|v_{||}|}\sqrt{g}B\right)\sum_{\substack{n,m\\n',m'}}\sum_{p=-\infty}^{\infty} e^{-i\frac{2\pi}{N_p}(n'-n)\varphi}\times$$

$$\times\left(\frac{\partial F_s}{\partial\epsilon}\right)_\mu \frac{\omega - 2\pi(\frac{n}{N_p}J - mI)\omega^*}{m\left\langle\omega_d^\vartheta\right\rangle + \frac{n}{N_p}\left\langle\omega_d^\varphi\right\rangle + \left\{\begin{matrix}\sigma(p+nq)\\ p\end{matrix}\right\}\omega_{\left\{\begin{matrix}t\\b\end{matrix}\right\}} - \omega} L^{(1)*}_{m'n'}\mathcal{M}^{m'n'*}_{pn}\, L^{(1)}_{mn}\mathcal{M}^{mn}_{pn}$$

definition of $\mathcal{M}^{m'n'}_{pn}$:

for passing particles:

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{i[2\pi(m'+n'q)\vartheta'' - (p+nq)\omega_t t'']}\right\rangle_{\vartheta''}$$

for reflected particles:

$$\mathcal{M}^{m'n'}_{pn} = \left\langle e^{2\pi i(m'+n'q)\vartheta''}\cos(p\omega_b t'')\right\rangle_{\vartheta''}$$

$\langle\ldots\rangle$ denotes the transit or bounce average

perturbed particle Lagrangian:

$$L^{(1)} = -(Mv_\|^2 - \mu B)\vec{\xi}_\perp\cdot\vec{\kappa} + \mu B\vec{\nabla}\cdot\vec{\xi}_\perp$$

Realization of kinetic MHD in CAS3D-K

CAS3D-K: perturbative stability code based on a hybrid MHD-drift kinetic model

- **3-dimensional**
- **general mode structure and equilibrium**
- **particle drifts are approximated as bounce averaged drifts**
- **zero radial orbit width and passing particles (at the moment)**
- **perturbative growth/damping rates from:**

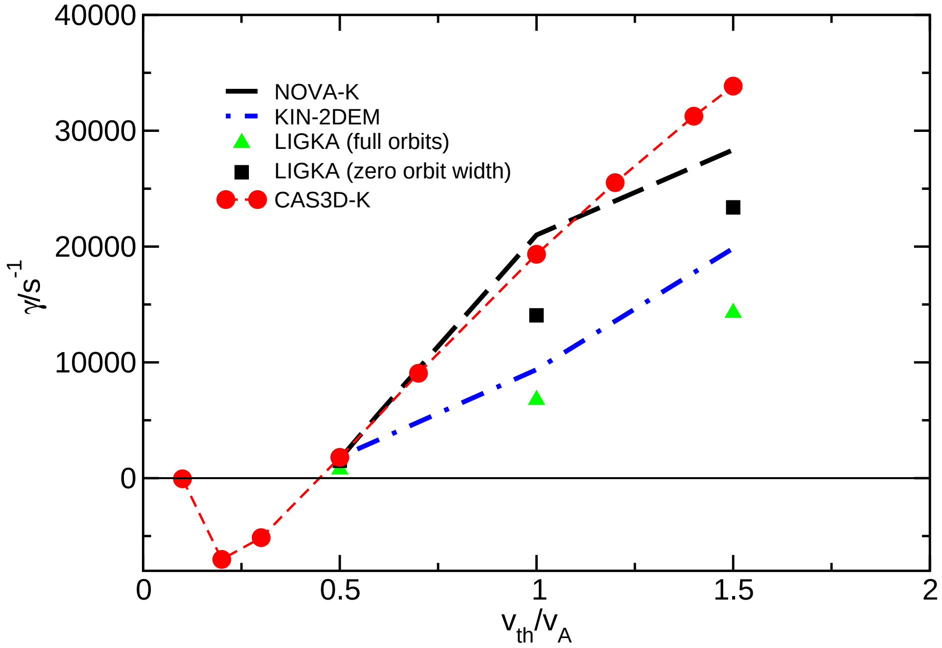
$$\Delta\omega_s + i\gamma_s \approx \frac{1}{2}\frac{\delta W_{\mathrm{s}}(\omega_0)}{\delta W_{\mathrm{mag}}}\omega_0$$

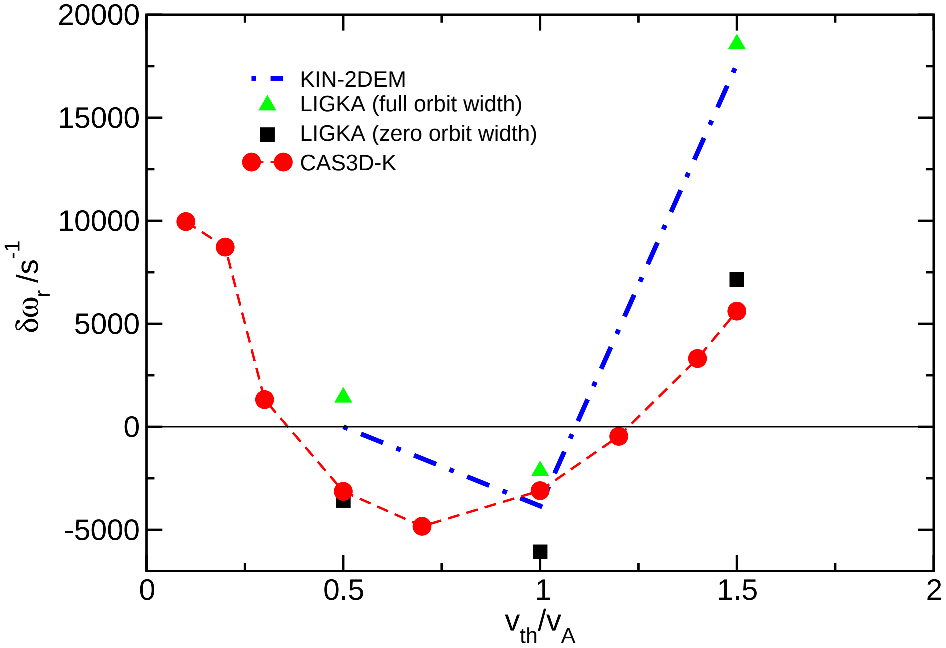
using the MHD eigenfunctions and the MHD frequency ω_0

- δW_{mag} **from the ideal MHD stability code** $CAS3D$ **(C. Nührenberg, 1996, 1998, 2000, . . .)**
- **note: comparable codes for 2D (e.g. NOVA-K with FOW and FLR)**

(3,-2)/(2,-2) TAE Benchmark with LIGKA

circular tokamak $A = 4$, Maxwellian distribution of fast hydrogen ions





LIGKA gyro-kinetic eigenvalue code (Ph. Lauber et al., J Comp. Phys. 2007)

3D analytical theory

valid in the limit of very localized modes and for an isotropic distribution of the hot particles (Kolesnichenko et al. PoP 2002)

hot particle growth rate:

$$\gamma = \frac{3\pi\beta_\alpha}{64k^2r^2} \sum_{\nu,\mu,j} \left|\epsilon^{(\mu\nu)}\right|^2 \frac{w \int_w^{w/\sqrt{\epsilon_{eff}}} du u (u^2 + w^2)^2 (\omega \partial/\partial u^2 + \omega_d) f_0}{\int_0^\infty du u^4 f_0}$$

with

$$w = \left| v_{A*} \left(1 + 2j \frac{\iota_* - \nu N}{\mu_0 \iota_* - \nu_0 N} \right) \right| / v_0 \qquad u = v/v_0$$

$$\iota_* = (2n + \nu N)/(2m + \mu_0) \qquad k = [(m+p)\iota - n + s] R_0^{-1}$$

3D analytical theory - What can we learn?

(see Kolesnichenko et al. PoP 2002)

- proportionality to equilibrium quantities

$$\frac{\gamma}{\omega_0} \propto A^2 \sum_{m'n'} |\epsilon^{\kappa}_{m'n'}|^2 \approx A^2 \sum_{m'n'} |\epsilon^{B}_{m'n'}|^2$$

- coupling is approximately given by the structure of B
  $\Rightarrow$ investigate spectrum of B
- note, that for a TAE in a large aspect ratio tokamak: $\frac{\gamma}{\omega_0}$ is independent of the equilibrium
- the resonance condition $\omega - k_{||} v_{th} = 0$ determines

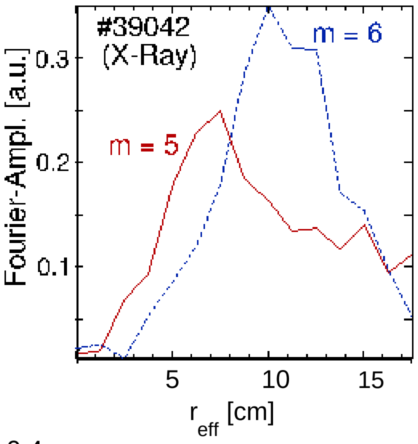
$$v^{\mathrm{res}}_{m'n'} = v_A \left| 1 \pm \frac{m'\iota^* + n' N_p}{m\iota^* + n} \right|^{-1}$$

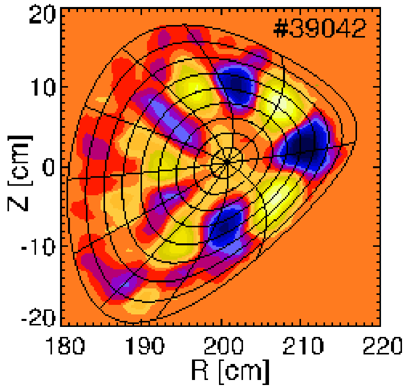
i.e. well known resonances at $v_0 = v_A$ and $v_0 = v_A/3$ for a Tokamak

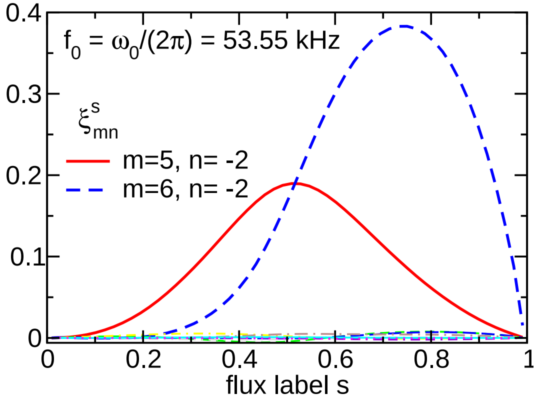
TAEs in W7-AS (#39042) and W7-X

W7-AS

A. Weller et al., Phys. Plasmas, **8**, 931 (2001):





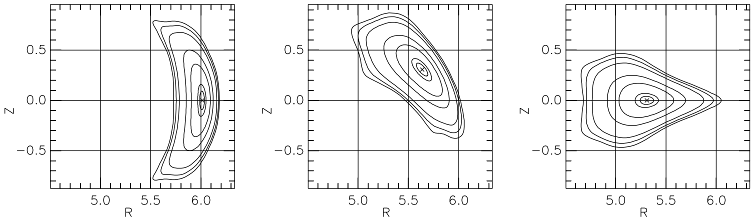


W7-X

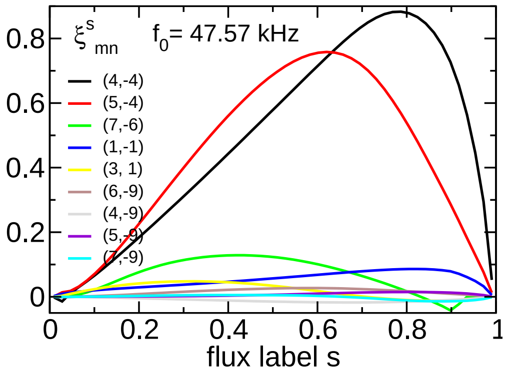
equilibrium:

M. Drevlak et al., Nucl. Fusion, 45, 731 (2005):

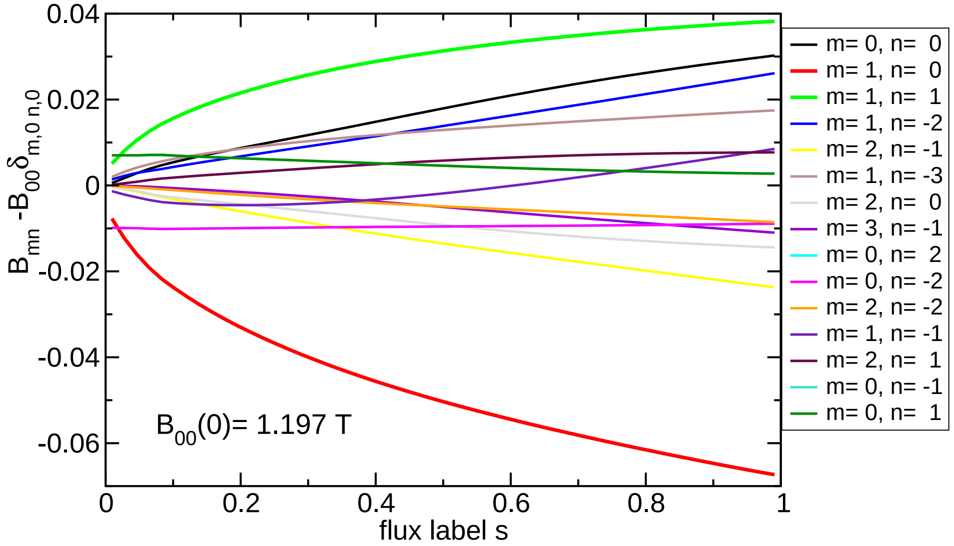
from PIES calculation: practically island free

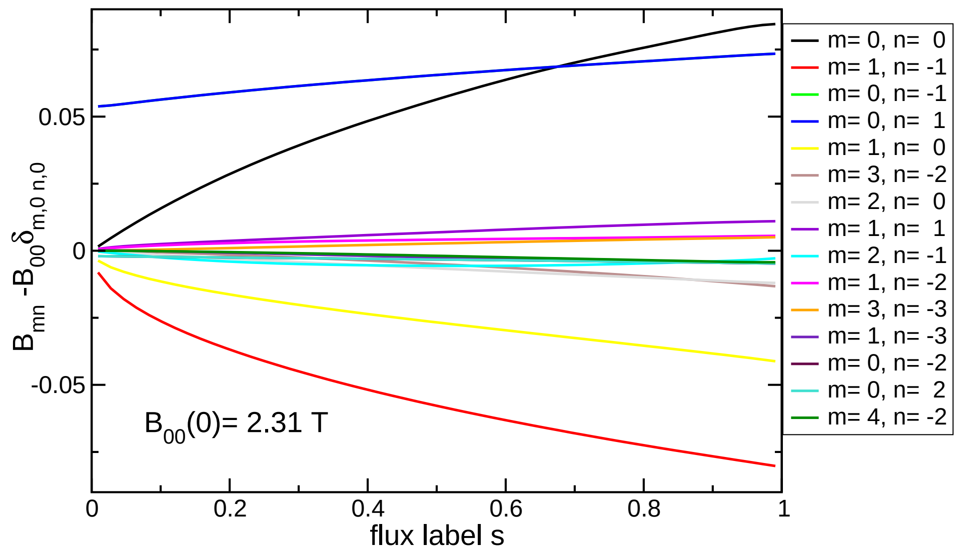


mode:



extract possible coupling from B spectrum

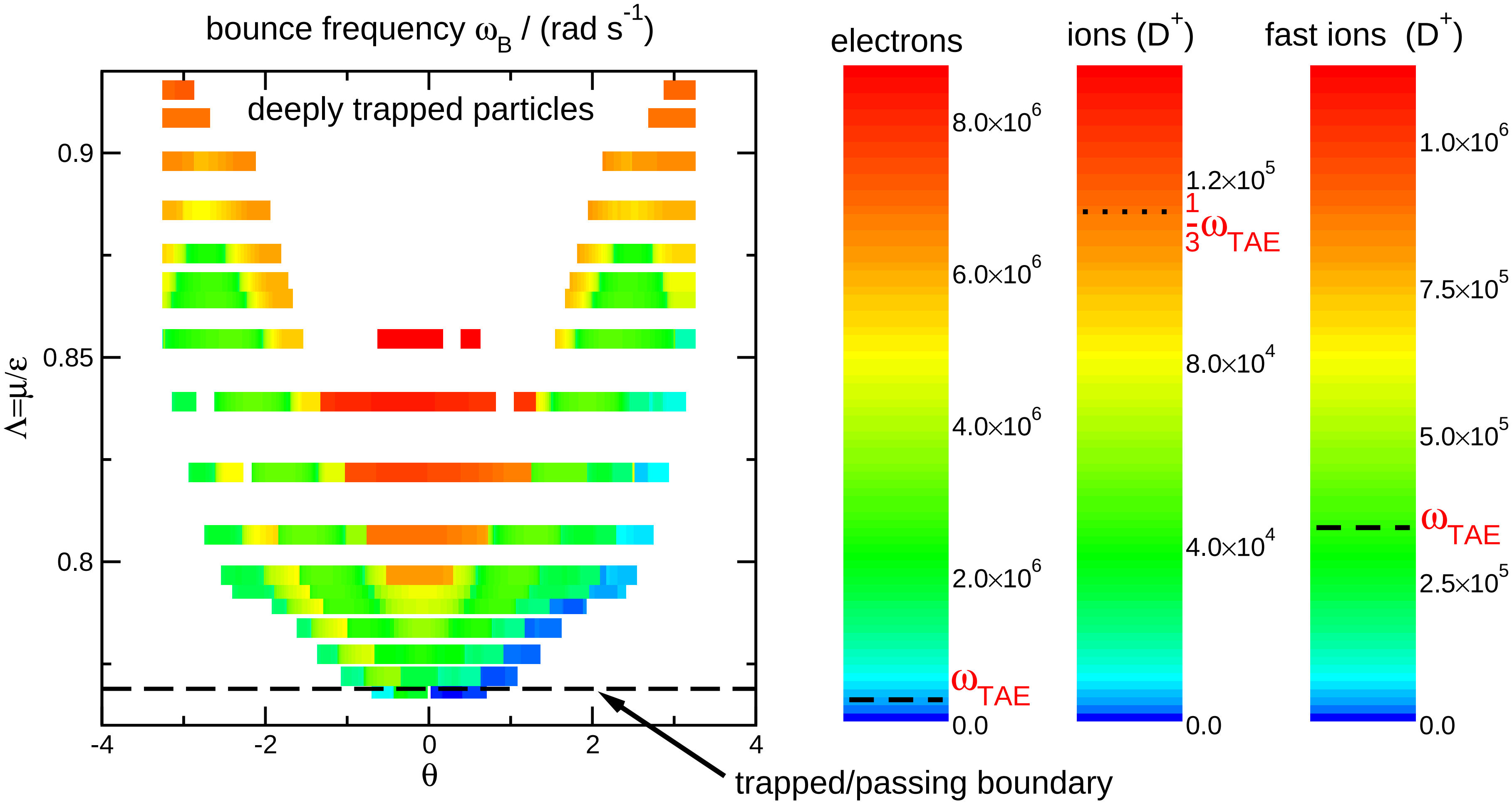




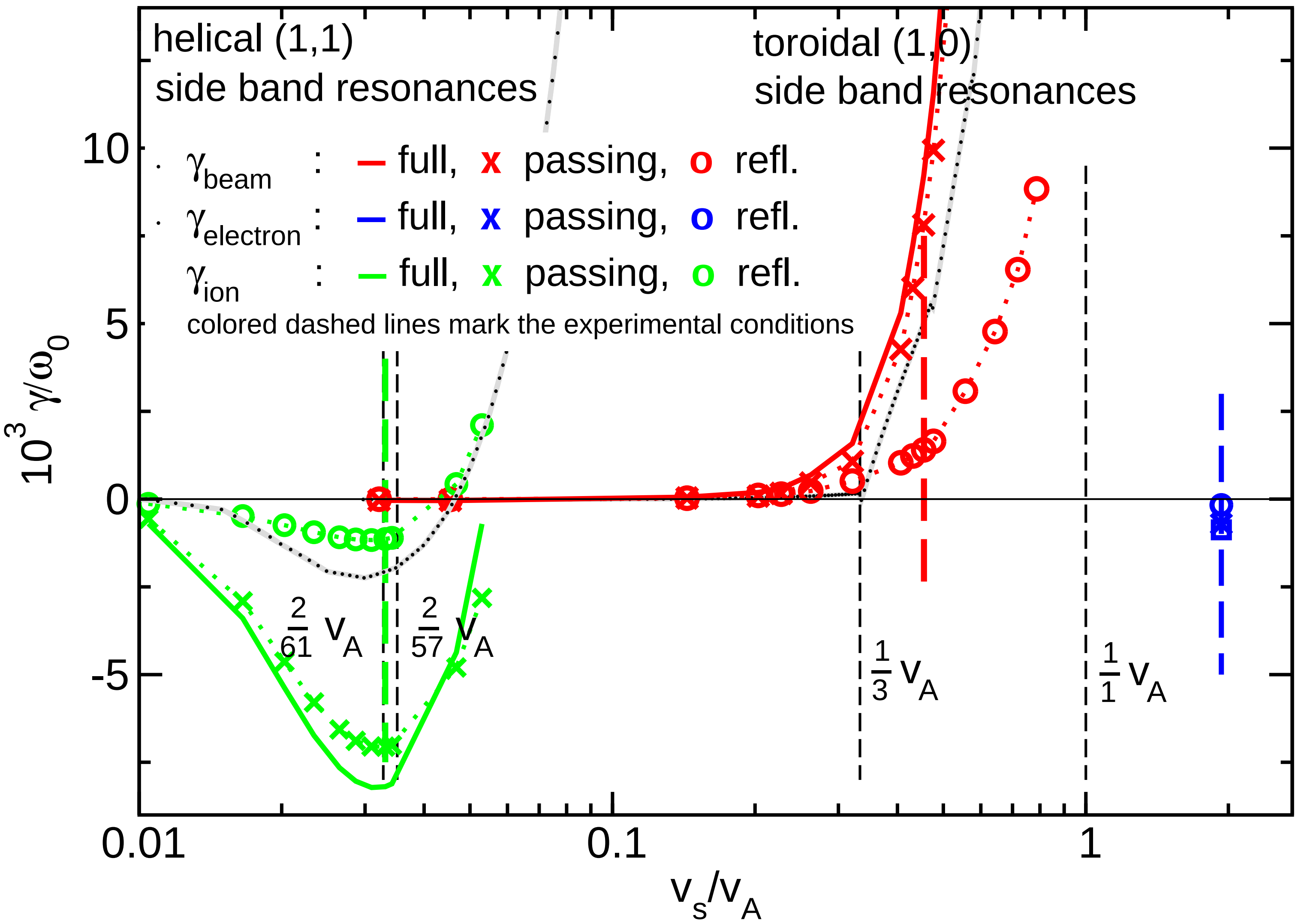
W7-AS

W7-X

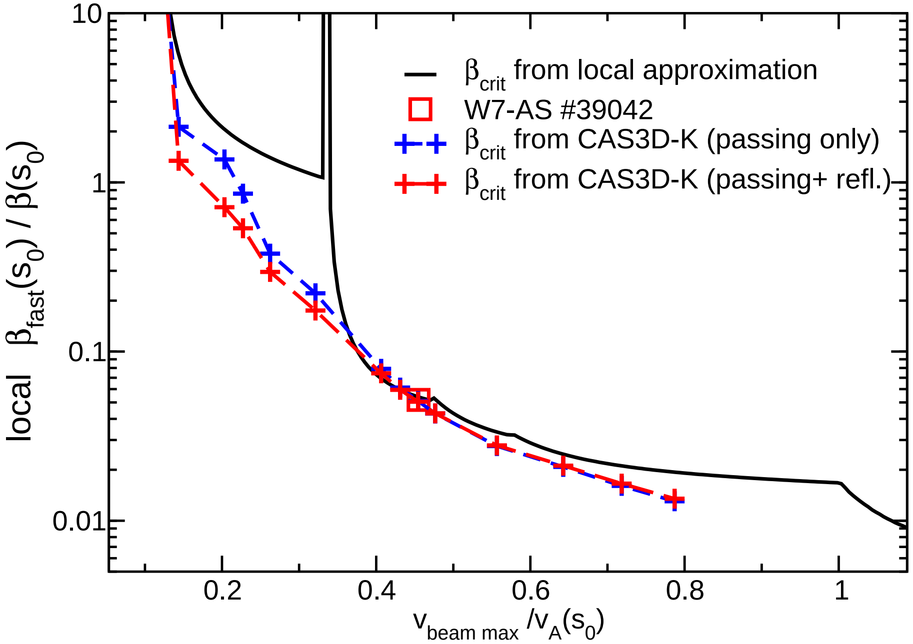
W7-AS: influence of reflected particles



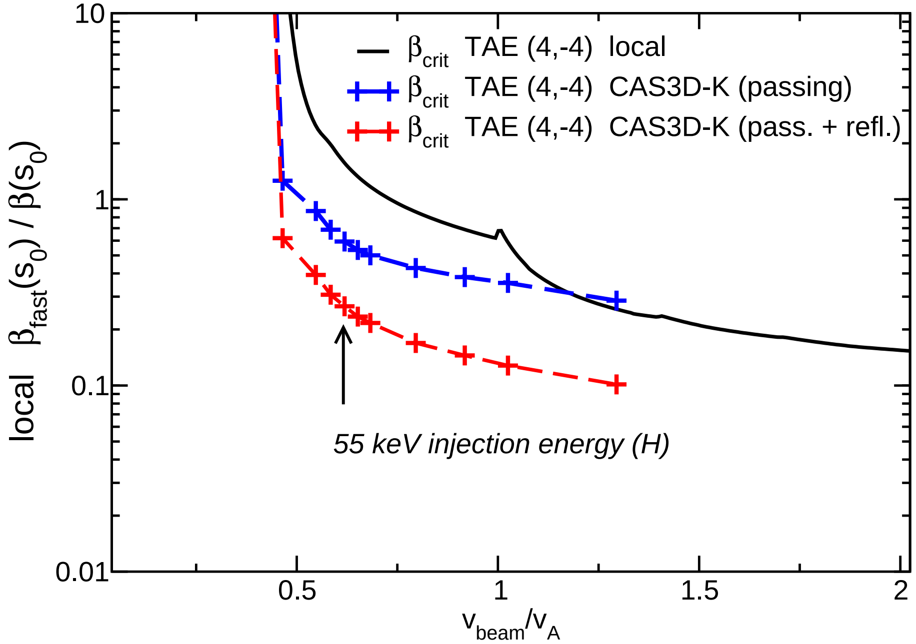
W7-AS: influence of reflected particles



stability diagrams/ critical β

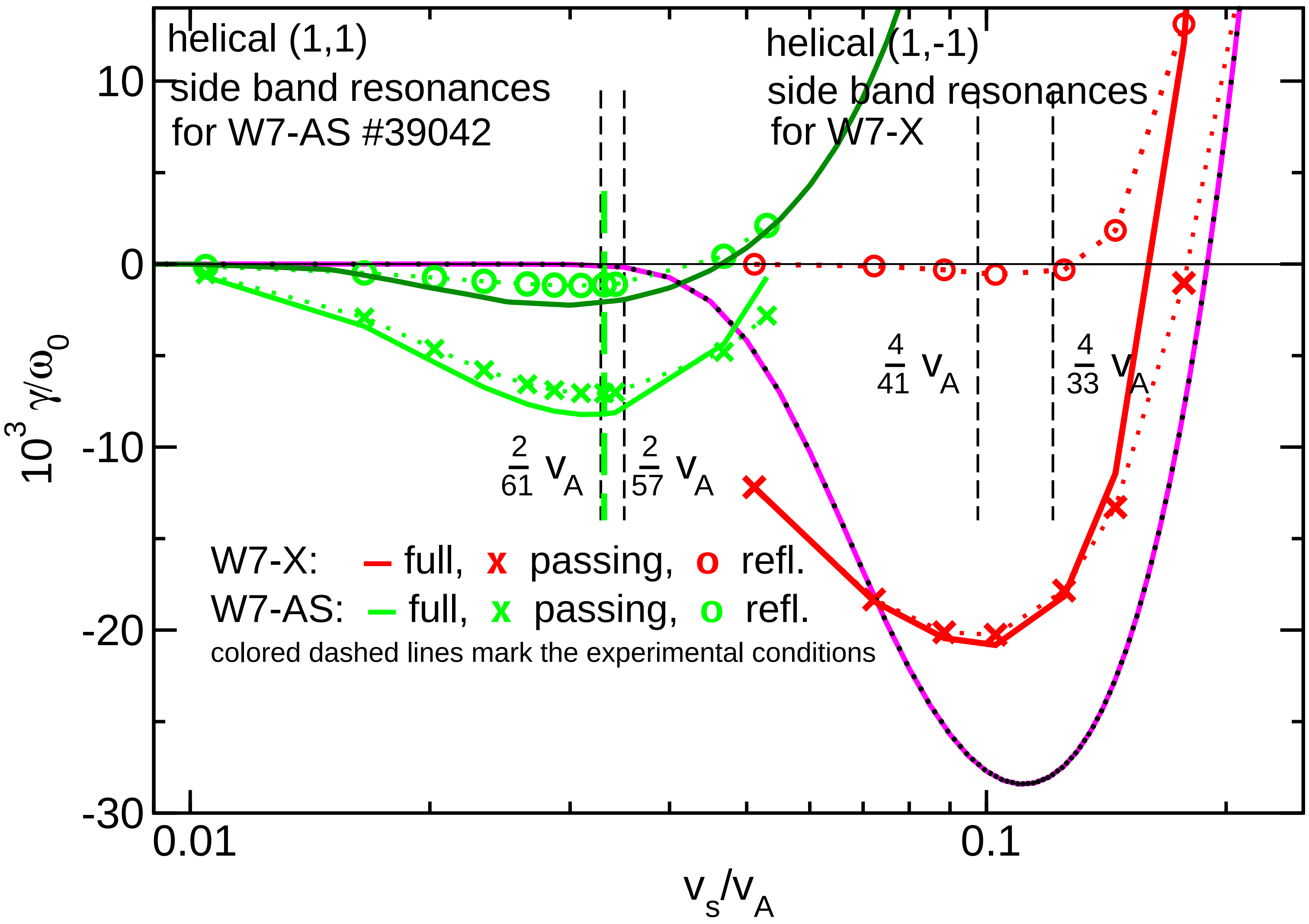


(5,-2),(6,-2) TAE in W7-AS



(4,-4),(5,-4) TAE in W7-X

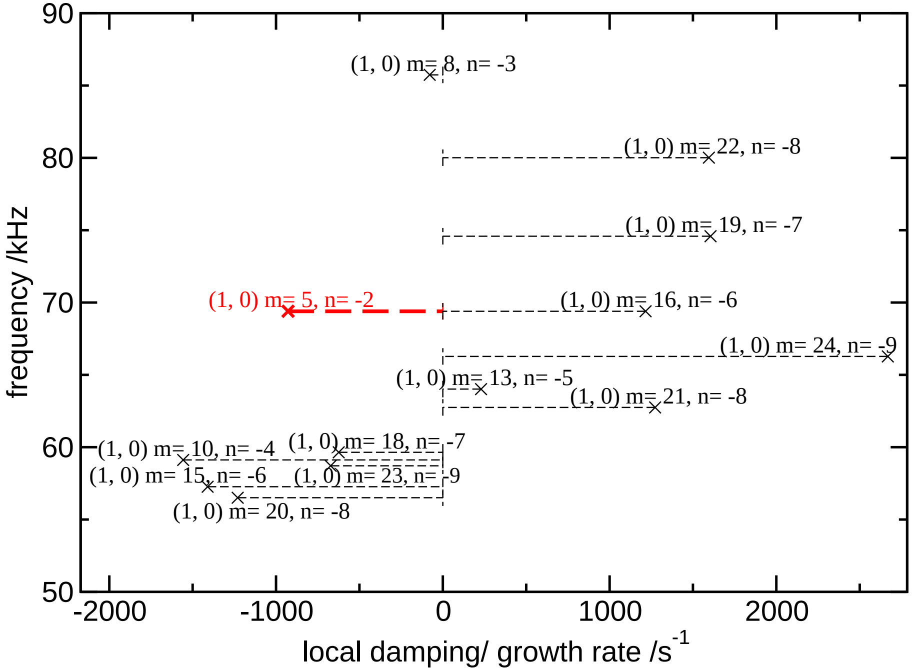
damping by thermal ions



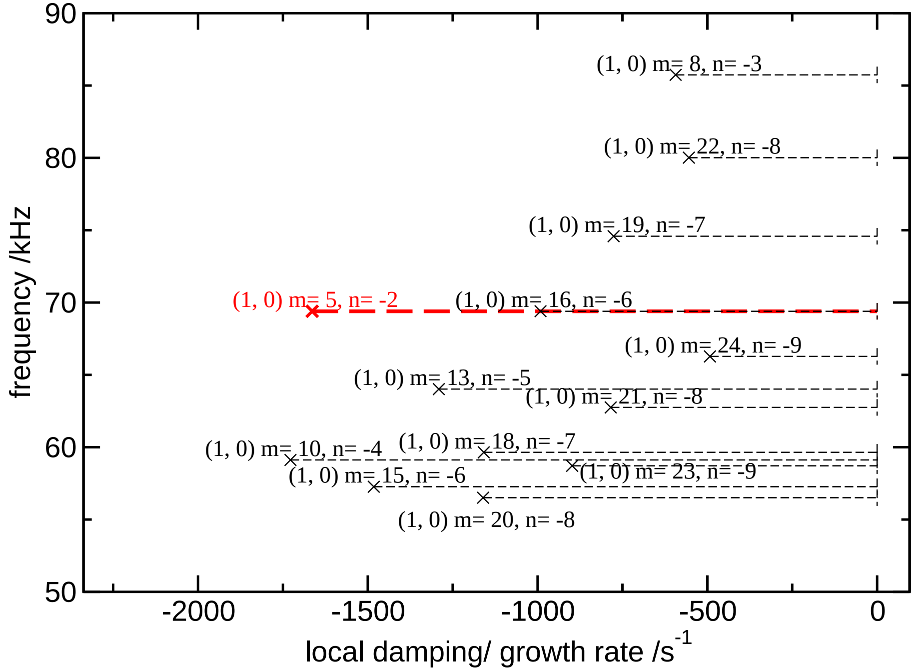
destabilization by temperature gradients

TAE mode frequencies and growth/ damping rates from a local computation

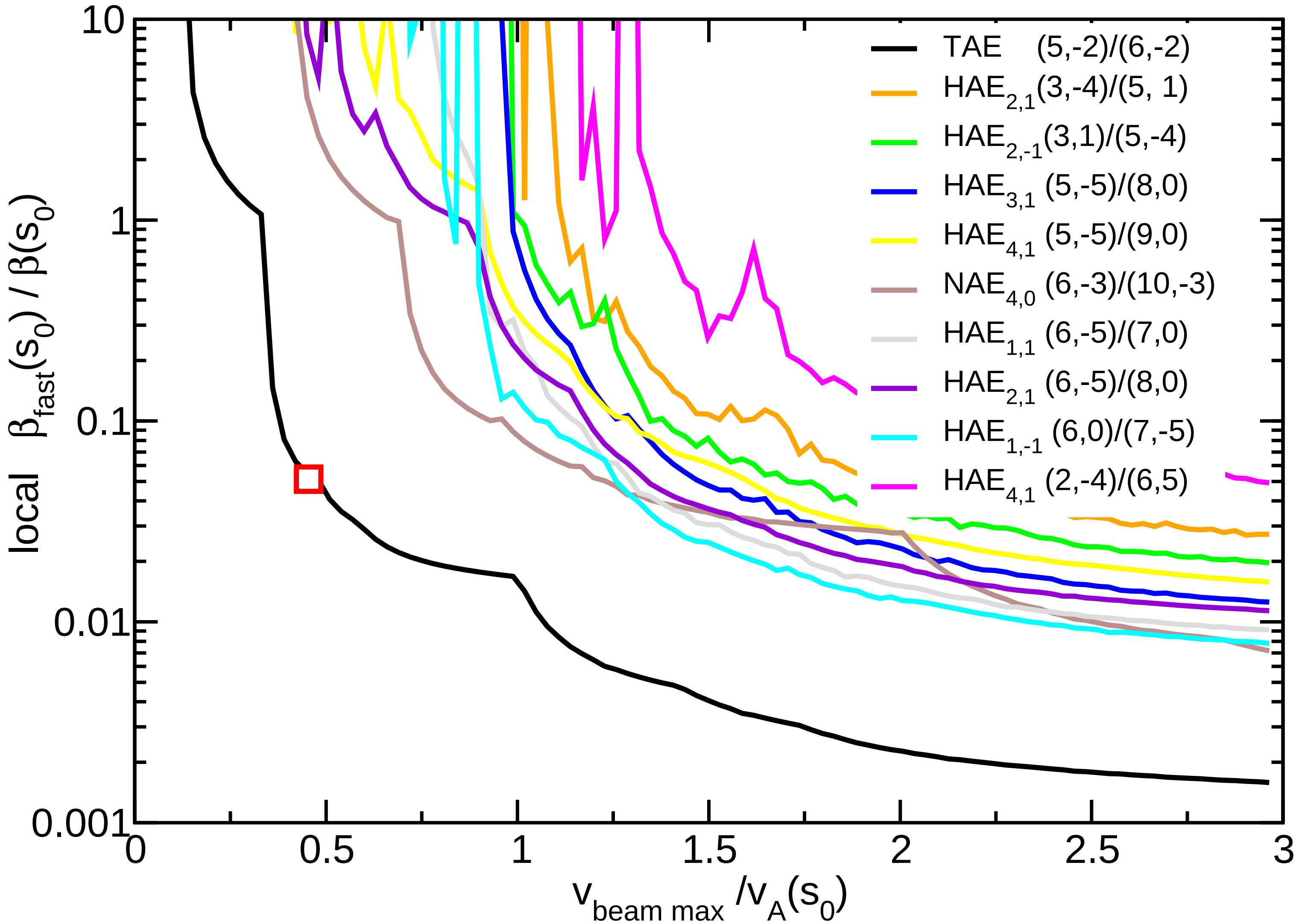
with a temperature gradient:



without a temperature gradient:



W7-AS: most unstable mode at given m (LGRO)



extending the model: reduced MHD + FLR/ finite $E_{||}$

$$\vec{\nabla}\cdot\vec{\nabla}_\perp\left((\frac{3}{4}\rho_i+\rho_s)\frac{\omega^2}{v_A^2}\vec{\nabla}\cdot\vec{\nabla}_\perp\phi\right)+\vec{\nabla}\left(\frac{\omega^2}{v_A^2}\vec{\nabla}_\perp\phi\right)=-\vec{\nabla}\cdot\left(\vec{b}\vec{\nabla}^2(\vec{b}\cdot\vec{\nabla})\phi\right)$$

$$-\vec{\nabla}\cdot\left(\vec{b}\vec{\nabla}\cdot\left(\frac{\mu_0 j_{||}}{B}\vec{b}\times\vec{\nabla}\phi\right)\right)+\nabla\left(\frac{\mu_0 i\omega p_\perp^{(1)}}{B^2}\vec{b}\times\vec{\nabla}B\right)+\nabla\left(\frac{\mu_0 i\omega p_{||}^{(1)}}{B}\vec{b}\times\vec{\kappa}\right)$$

see e.g. Rosenbluth et al., Strauss, Qin et al. . . .

fast particles can be coupled to the equation via the pressure pertubation

realization:
either eigenvalue code
(like NOVA-K, CASTOR-K, CAS3D3-K ...)
or
to an initial value problem as in HAGIS replacing its external MHD input

numerical realization: eigenvalue problem

Code for Kinetic Alfvén waves

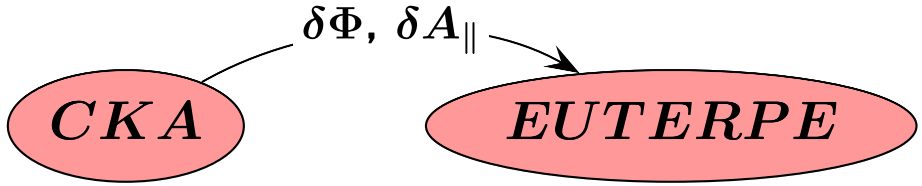
- **finite elements (B-splines) in all three directions**
- **B-splines of arbitrary order and spacing**
- **parallel and serial code version**
- **parallel: SLEPC (Scalable Library for Eigenvalue Problem Computation, based on PETSC, Hernandez et al., Unversidad de Valencia)**
- **iterative solvers on SLEPC: power, Arnoldi, subspace, . . .**

CKA-EUTERPE Hybrid Code by T. Fehér

- **Global Alfvén eigenmodes could be driven unstable by energetic particles**
- **Numerical tool for perturbative stability analysis in 3D geometry**

Hybrid model

code:



	CKA	EUTERPE
describes:	**Alfvén waves**	**Energetic particles**
solves:	**MHD EQs**	**Gyrokinetic EQs**
method:	**Spline decomposition**	**PIC**

Energy transfer between the particles and the wave:

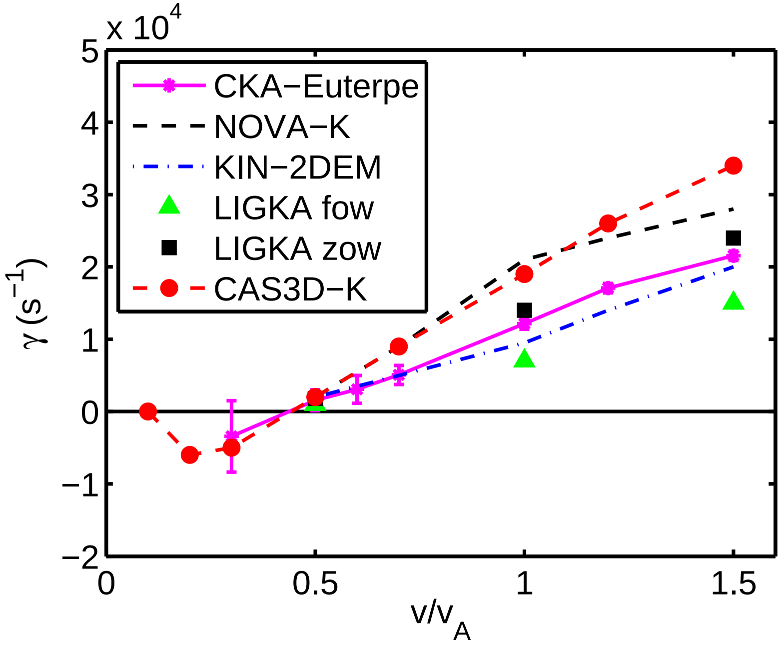
$$\gamma = \frac{1}{2\mathcal{E}_{field}} \frac{\partial \mathcal{E}_{field}}{\partial t} = -\frac{1}{2\mathcal{E}_{field}} \frac{\partial \mathcal{E}_{kin}}{\partial t} = -\frac{1}{2\mathcal{E}_{field}} \int j \cdot \mathbf{E} \mathrm{d}^3 r$$

CKA-EUTERPE - Tokamak benchmarks

- Energetic particles with density gradient
- Stability of global Alfvén modes are calculated

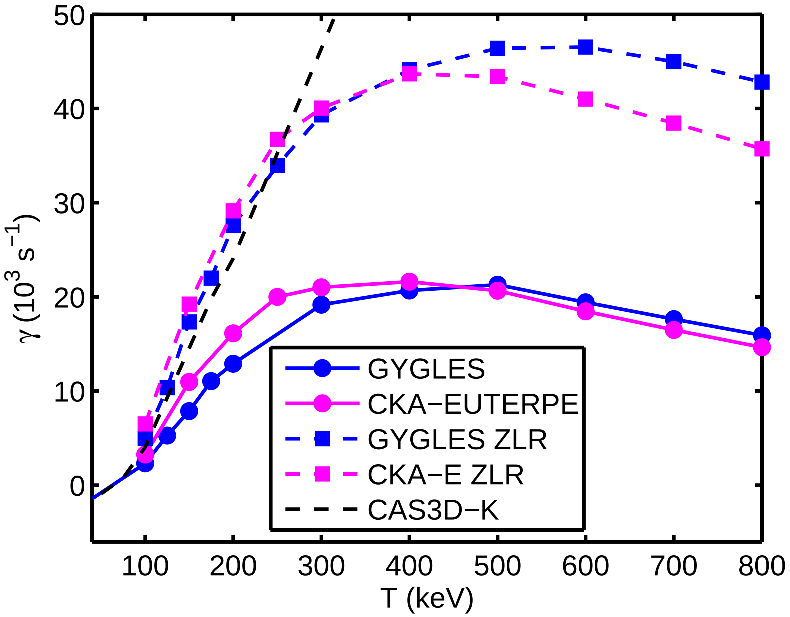
Benchmark 1

R=4m, a=1m, B=3T, n_0=5·10^{19}
TAE mode (n=-2, m=2,3)



GYGLES benchmark

R=10m, a=1m, B=3T, n_0=2·10^{19}
Stability of TAE (n=-6, m=10,11)



Summary

- fully gyro-kinetic global linear electro-magneticsimulation possible with a PIC method (GYGLES) damping rates, mode structure of Alfvén modes
- many cases seem to be modified TAE rather than EPM
- kinetic MHD is valuable tool in 2D/3D

- standard for 3D: CAS3D-K (zero orbit width)
- two drift-kinetic MHD perturbative hybrid codes (VENUS(+CAS3D) and AE3D) and . . .
- one gyro-kinetic MHD perturbative hybrid code (CKA/ EUTERPE) under development
- investigation of full orbit width and finite gyro radius effcts under way (see: IAEA TM Austin, September 2011)

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